Foreign Ownership of U.S. Safe Assets: Good or Bad?*

Jack Favilukis — UBC
Sydney C. Ludvigson — NYU and NBER
Stijn Van Nieuwerburgh — NYU NBER CEPR

First draft: February 10, 2011
This draft: September 8, 2014

Abstract

The last 20 years have been marked by a sharp rise in international demand for U.S. reserve assets, or safe stores-of-value. What are the welfare consequences to U.S. households of these trends, or of a reversal? In a lifecycle model with aggregate and idiosyncratic risks, the young and oldest households may benefit substantially from such capital inflows, but middle-aged savers may suffer from greater exposure to systematic risk in equity and housing markets. Under the veil of ignorance, a newborn in the lowest wealth quantile is willing to forego 2.7% of lifetime consumption to avoid a large capital outflow. JEL: G11, G12, E44, E21

*Favilukis: Department of Finance, University of British Columbia Sauder School of Business; Email: jack.favilukis@sauder.ubc.ca; Tel: (604) 822-9414 http://www.sauder.ubc.ca/Faculty/People/Faculty_Members/Favilukis_Jack. Ludvigson: Department of Economics, New York University, 19 W. 4th Street, 6th Floor, New York, NY 10012; Email: sydney.ludvigson@nyu.edu; Tel: (212) 998-8927; http://www.econ.nyu.edu/user/ludvigsons/. Van Nieuwerburgh : Department of Finance, Stern School of Business, New York University, 44 W. 4th Street, 6th Floor, New York, NY 10012; Email: svnieuwe@stern.nyu.edu; Tel: (212) 998-0673; http://pages.stern.nyu.edu/ svnieuwe/. We are grateful to Mark Aguiar, Pedro Gete, Gita Gopinath, Pierre-Olivier Gourinchas, Tarek Hassan, Jonathan Heathcote, Bernard Herskovic, Matteo Maggiori, Jaromir Nosal, Helene Rey and to seminar participants at the NBER conference on Sovereign Debt and Financial Crises, November 2013, the conference on International Capital Flows and Spillovers, December 2012, the CEPR Developments in Macroeconomics and Finance Conference, November 2012, the BU/Boston Fed Conference on Macro-Finance Linkages, November 2012, NBER International Finance and Macroeconomics meeting March 2012, the Frontiers of Macroeconomics Conference at Queens University April 2012, at Carnegie Mellon, Columbia University, New York University, Ohio State University, Southhampton University, University of Miami, University of Southern California, University of Pennsylvania Wharton School and UC Davis for helpful comments. Any errors or omissions are the responsibility of the authors.

1 Introduction

The last 20 years have been marked by a sharp rise in international demand for U.S. reserve assets, or safe stores-of-value. This has led to an unprecedented degree of foreign ownership of U.S. government and government-backed debt, most of it held by Foreign Official Institutions such as central banks. In 1994, foreign holdings of U.S. Treasuries amounted to 17% of marketable Treasuries outstanding. By the end of 2008, foreigners owned 51% of all U.S. federal government debt.[1] These trends have raised questions about the sustainability of large "global imbalances" between the demand for and supply of U.S. reserve assets, and they have invited speculation over the possible economic consequences of a sell-off of U.S. debt by foreign governments.[2]

An important aspect of these trends is that foreign demand for U.S. Treasury securities has been dominated by Foreign Official Institutions (FOIs), namely foreign governmental entities such as central banks. Because these institutions face political, legal, and regulatory restrictions on the types of assets they can hold, their motivations for saving are quite different from those of private investors. FOIs take extremely inelastic positions in U.S. safe assets, implying that when they receive funds to invest, they buy U.S. Treasuries regardless of price (Krishnamurthy and Vissing-Jorgensen (2007)). Foreign official flows have also been found to be the main driver of uphill capital flows, which are not well described by two-country neoclassical models of private optimizing agents (Alfaro, Kalemli-Ozcan, and Volosovych (2011)). Likewise, Aguiar and Amador (2011) emphasize that uphill flows are mainly driven by government net assets, not private flows.[3] Indeed, the persistent and growing U.S. trade deficits since 1994 have been financed almost exclusively by an upward trend in net foreign holdings by foreign governments of U.S. assets considered to be safe stores-of-value. By contrast, net foreign holdings of risky securities have fluctuated near

[1] China is the largest such owner, holding 26%, as of December 2010, of all tradable U.S. Treasury and Agency debt, followed by Japan (20%), the major banking centers (Caribbean, Luxembourg, UK, Ireland, Belgium, 14%), and the "rest of Asia" (Hong Kong, Singapore, Korea, India, Malaysia, Philippines, 12%). Data source: the U.S. Treasury Department, Treasury International Capital System.

[2] See for example, Obstfeld and Rogoff (2009), Bernanke (2011) and Fahri, Gourinchas, and Rey (2011).

[3] Aguiar and Amador (2011) provide a potential explanation whereby governments of developing nations use saving abroad as a means of overcoming limited commitment to defaulting on external debt and expropriating foreign investment positions.

zero.

Despite a vigorous academic debate on the question of whether global imbalances are a fundamentally benign or detrimental phenomenon,[4] little is known about the potential welfare consequences of foreign governmental ownership of U.S. safe assets. This paper analyzes the welfare consequences of these flows for U.S. households. We argue here that a complete understanding of the welfare implications requires a model with realistic heterogeneity, life-cycle dynamics, and plausible financial markets. We study a two-sector model of housing and non-housing production where heterogeneous agents face limited opportunities to insure against idiosyncratic and aggregate risks. A crucial source of aggregate risk in the model is a shock to foreign ownership of the domestic riskless bond, calibrated to match U.S. data. This shock affects asset values and welfare because it alters the effective supply of safe assets available to domestic households.

The model economy we study implies that foreign purchases (or sales) of the safe asset can have quantitatively large distributional consequences, reflecting sizable tradeoffs between generations, and between economic groups distinguished by wealth and income. The implications for domestic welfare are heavily influenced by the endogenous response of asset markets to fluctuations in foreign holdings of the safe asset. Foreign purchases of the safe asset act like a positive economic shock and have an economically important downward impact on the risk-free interest rate, consistent with empirical evidence.[5] But although lower interest rates boost output, equity and home prices, foreign purchases of the domestic riskless bond also reduce the effective supply of the safe asset, thereby exposing domestic savers to greater systematic risk in equity and housing markets. In response, risk premia on housing and equity assets rise, substantially (but not fully) offsetting the stimulatory impact of lower interest rates on home and equity prices.

These factors imply that the young and the old experience welfare gains from a capital inflow, while middle-aged savers suffer. The young benefit from higher wages and from lower interest rates, which reduce the costs of home ownership and of borrowing in anticipation of

[4] See Mendoza, Quadrini, and Rios-Rull (2009), Caballero, Fahri, and Gourinchas (2008a), Caballero, Fahri, and Gourinchas (2008b), Obstfeld and Rogoff (2009), and Caballero (2009).

[5] See Krishnamurthy and Vissing-Jorgensen (2007), Warnock and Warnock (2009), and Bernanke (2011).

higher expected future income. But middle-aged savers are hurt because they are crowded out of the safe bond market and exposed to greater systematic risk in equity and housing markets. Although they are partially compensated for this in equilibrium by higher risk-premia, they still suffer from lower expected rates of return on savings. By contrast, retired individuals who are drawing down assets at the end of life experience a significant net gain from even modest increases in asset values that accompany a capital inflow.

The magnitude of these effects for some individuals is potentially large. In the highest quintile of the distribution of external leverage, the youngest working-age households would be willing to give up about 0.2% of life time consumption in order to avoid just one year of a typical annual decline in foreign holdings of the safe asset. This effect could be several times larger for a greater-than-typical decline, and many times larger for a series of annual declines in succession or spaced over the remainder of the household's lifetime. Under the "veil of ignorance," newborns typically benefit from foreign purchases of the safe asset, unless they are wealthy. Newborns at the 25th percentile of the net worth distribution would be willing to forgo up to 2.7% of lifetime consumption in order to avoid a large capital outflow; a median newborn benefits by less, while a newborn in the 75th percentile slightly prefers outflows.

The nature of these results is closely related to the richness of the domestic model economy, in at least three ways. First, because time-varying risk premia play a key role in how asset values respond to capital flows, it is important that the model has both plausible heterogeneity and a non-trivial portfolio choice between risky and safe assets, which requires modeling aggregate risk. Second, because domestic households (in aggregate) could undo much of the impact of foreign purchases on U.S. interest rates by altering their saving behavior, it is important that domestic agents in the model optimally choose bond holdings, rather than taking interest rates as exogenous. Third, because foreign flows have important effects on collateral values and borrowing terms, it is important to explicitly account for housing. For example, capital inflows could be welfare reducing for young households who are first-time home buyers if they are forced to purchase assets at greatly elevated prices. This channel is outweighed in our framework for all but the wealthiest households by the improved insurance opportunities that accompany an inflow, through the endogenous responses of both the housing risk premium and housing supply, which together offset the stimulatory impact of lower interest rates on home prices and limit the extent to which they can rise. All of these factors have important effects on welfare. While some model ingredients could be dispensed with, our aim is to provide a quantitative welfare analysis, rather than a set of qualitative results. A rich model is indispensable for our purposes.

This paper is related to the literature on global imbalances in international capital markets and, more loosely, to the literature on Sudden Stops, which studies reversals of international capital flows in emerging economies.[6] Caballero, Fahri, and Gourinchas (2008a), Caballero and Krishnamurthy (2009) (discussed further below), and Mendoza (2010) study the economic consequences of capital inflows in representative agent economies, but do not study the welfare outcomes of these flows. A premise of this paper is that a complete understanding of the welfare implications requires a model with reasonable heterogeneity, life-cycle dynamics, and plausible financial markets.

Our model is silent on the economic implications of gross flows, and we do not study cyclical fluctuations in the value of net foreign holdings of other securities which, unlike net foreign holdings of U.S. safe assets, show no upward trend. By contrast, Gourinchas and Rey (2007) and Maggiori (2011) investigate how the net foreign asset position of the U.S. invested in risky securities varies cyclically across normal and "crisis" times, as well as how gross flows are affected. But these papers are silent on the reasons for the large and growing net foreign debtor position of the U.S. in good times, and on its upward trend over time. We view these studies as complementary to ours.

Our model is also silent on the welfare consequences of the change in foreign holdings of U.S. safe assets for the rest of the world. A complementary literature focuses on the effects in the countries from which the flows originate. The implications for both domestic and foreign households of global imbalances have been studied in models that are less rich on the domestic side than the economy studied here, e.g., in models without aggregate risk (Mendoza, Quadrini, and Rios-Rull (2009)), or in models without household/worker heterogeneity (Aguiar and Amador (2011)). We argue here that both heterogeneity and

[6] The application of this paper is to the developed economy of the United States. For a classification of Sudden Stops in emerging economies, see Calvo, Izquierdo, and Talvi (2006).

aggregate risk are central to the welfare implications of these flows. These papers also generate foreign capital flows as endogenous outcomes of a model with trade. Our focus here is different. Rather than attempting to model the mechanics of the political economy and trade adjustment that generate the right pattern of FOI flows, we take the observed flows as given and study the implications for U.S. welfare. A benefit of this approach is that we can study a much richer model of the domestic economy, with clearer welfare implications for U.S. households. A limitation is that we cannot make statements about global welfare.

The model in this paper builds on the incomplete markets model studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2008) (FLVN). FLVN do not study the welfare consequences of international capital flows, as here. This requires introducing an additional source of aggregate uncertainty, namely a shock to foreign holdings relative to output, which cannot be insured away. This additional source of aggregate risk, which adds two new state variables over which agents must form expectations, substantially complicates the model of FLVN but has important implications both for asset markets and welfare.

The rest of this paper is organized as follows. The next section discusses the recent history of foreign purchases of U.S. government securities, and how we model them. Section 3 describes the model, including the dynamics of foreign holdings of domestic bonds, the equilibrium, the welfare measures, and the calibration. Section 4 presents the results, focusing on the macroeconomic, asset market, and welfare consequences of fluctuations in foreign ownership of the domestic safe asset. Section 5 summarizes and concludes.

2 Modeling Recent Trends in Safe Asset Flows

This section provides a brief summary of some of the most salient features of recent trends in international capital flows to the U.S. We refer the reader to Alfaro, Kalemli-Ozcan, and Volosovych (2011) and Favilukis, Kohn, Ludvigson, and Van Nieuwerburgh (2013) for a more detailed discussion. We define *net foreign holdings* of U.S. assets, or alternatively, as the U.S. *net liability position* as the value of foreign holdings of U.S. assets minus U.S. holdings of foreign assets. Figure 1 panel A shows that foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from \$200 billion in 1984, or 14.6% of marketable Treasuries outstanding, to \$3.25 trillion in 2008, or 51% of marketable Treasuries. Foreign Treasury holdings further grow to \$5.6 trillion at the end of our sample in June 2013. Foreign holdings of U.S. agency and Government Sponsored Enterprise-backed mortgage securities (referred to as *Agency debt* hereafter) quintupled between 2000 and 2007, rising from \$330 billion to \$1.4 trillion, or from 8% to 21% of total agency debt. Foreign holdings of U.S. Treasury (short- and long-term) and long-term Agency debt as a fraction of trend GDP more than doubled from 13.7% to 29.2% over the period 2000-2008 and stands at 39.4% at the end of our sample in 2013Q2.

Panel B of Figure 1 shows the fraction of foreign holdings relative to trend U.S. gross domestic product (GDP) over time. The figure reports both the raw series, as well as a series adjusted in 2009-2013 for the increase in the quantity of Treasury debt outstanding that occurred in those years as a result of the American Recovery and Reinvestment Act of 2009. The adjusted series equals the level of foreign holdings as a fraction of trend GDP that would have occurred in 2009-2013 had Treasury debt outstanding as a fraction of trend GDP been fixed at its 2008 level. For the unadjusted series, foreign holdings almost tripled from 2001 to 2010, increasing from 13.5% of trend GDP in 2001 to 39.4% by June 2013. But the adjusted series implies that foreign holdings were just 22.7% of trend GDP in June 2013, 16.7% lower than the unadjusted figure. This suggests that an unwinding of foreign holdings, at least relative to trend GDP, may have been underway by the end of our sample.

This paper is concerned with changes in capital flows that result from changes in the net foreign holdings of U.S. *safe* assets, which we define to be U.S. Treasury and Agency debt. Figure 2 shows that net foreign holdings of other securities as a fraction of U.S. Trend GDP have hovered close to zero since 1994, even as net foreign holdings of safe securities have soared. Thus *all* of the upward trend in net foreign holdings of U.S. securities since 1994 has been the result of an upward trend in net foreign holdings of U.S. safe assets. Indeed, although not shown in the graph, all of the upward trend in the overall U.S. net debtor position (which accounts for non-security assets such as Foreign Direct Investment) over the last 15 years is attributable to foreign purchases of U.S. safe assets.[7]

[7] Our model includes only two securities that could be traded: stocks and bonds. Thus, we calibrate our international capital flows to changes in flows on total financial securities. Other assets in the U.S. balance of

The rise in net holdings of U.S. safe assets by foreigners over time has coincided with downward trend in real interest rates. The real annual interest rate on the 10-year Treasury bond fell from 3.87% at the start of 2000 to 2.04% by the end of 2005, while the 10-year Treasury Inflation Protected (TIPS) rate fell from 4.32% to 2.12% over this period. Real rates fell further to all time lows during the economic contraction that followed. The real 10-year Treasury bond rate declined from 2.04% to -0.04% from 2006.12 to 2012.12, while the TIPS rate declined from 2.25% to -0.76%.[8]

Foreign official institutions have dominated these trends. In June 2010, according to data from the Treasury International Capital Reporting System (TIC), FOIs held 75% of all foreign holdings of U.S. Treasuries, a likely under-estimate, since some prominent foreign governments purchase U.S. securities through offshore centers and third-country intermediaries, purchases that would not be attributed to foreign official entities by the TIC system (Warnock and Warnock (2009)). FOI holdings account for a similarly large fraction of the *increase* in foreign holdings of Treasuries over time, especially in the last 10 years: they account for 81% of the increase in foreign ownership of U.S. Treasuries from March 2000 to June 2010. Over the longer time frame shown in Figure 2 (December 1994 to June 2010), FOI holdings account for 77% of the increase in foreign held Treasuries and 73% of the increase in Agency debt.

Official flows behave quite differently from private flows. Alfaro, Kalemli-Ozcan, and Volosovych (2011) find that official flows are the main driver of uphill capital flows and global imbalances and, together with Aguiar and Amador (2011), they argue that official flows are not well described by two-country neoclassical models with private optimizing agents (private flows are, but they go downhill). Kohn (2002) emphasizes that government entities have specific regulatory and reserve currency motives for holding U.S. Treasuries and face both legal and political restrictions on the type of assets that can be held, forcing them

payments system include foreign direct investment, U.S. official reserves, and other U.S. government reserves. Net foreign holdings on these assets also display no discernable upward trend since 1994. See Favilukis, Kohn, Ludvigson, and Van Nieuwerburgh (2013) for a detailed discussion of recent trends in international capital flows.

[8]To compute the real interest rate, we use the 10-year constant maturity Treasury rate minus realized inflation. We obtain similar results when we use the expectations of the average annual rate of CPI inflation over the next 10 years from the Survey of Professional Forecasters, in percent per annum (sources: U.S. Treasury, Survey of Professional Forecasters).

into safe securities. Historically (and in stark contrast to private investors), FOIs hold very small fractions of their portfolios in risky securities of any kind,[9] and Krishnamurthy and Vissing-Jorgensen (2007) report that demand for U.S. Treasury securities by governmental holders is, unlike private holders, extremely inelastic, implying that when these holders receive funds to invest they buy U.S. Treasuries, regardless of their price.

These observations suggest that it is appropriate to model foreign safe asset holdings as owned by governmental holders who inelastically place all of their funds in the domestic riskless bond. This can be accomplished by taking the observed flows as equilibrium outcomes, and calibrating the model's changes in net capital flows on safe assets to match those observed in data. We then feed these changes into the bond market clearing condition that determines the equilibrium interest rate on riskless bonds.[10] We now discuss our specification for net capital flows to U.S. safe assets.

Let $B_{F,t}$ denote the stochastic supply of foreign capital to the domestic bond market, i.e., $B_{F,t} > 0$ represents a net positive bond position by foreign holders (a net liability for domestic households). Given the timing convention of the budget constraint (5) below, $B_{F,t+1}$ is beginning of period $t+1$ debt and therefore known at time t. A positive net foreign asset inflow is identically equivalent to a trade deficit (negative trade balance), which is reflected in the aggregate resource constraint of the economy–see equation (9) below. Given a probability law for stochastic foreign holdings, households form beliefs about their evolution.

Let $\overline{Y}_t$ denote trend GDP. In the model, all aggregate variables grow deterministically at rate g, thus trend output is normalized to $\exp(gt)$. In the data, we use the Hodrick and Prescott (1997) (HP) filter to compute the trend component of GDP. We assume that households form beliefs according to a stochastic process for foreign holdings relative to trend

[9] In 2010, they held only 12% of their portfolio in risky securities. Source: Foreign Portfolio Holdings of U.S. Securities as of June 30, 2013, Department of the Treasury.

[10] The model below assumes that domestic and foreign inputs are perfectly substitutable, so that adjustments to the capital account don't effect total factor productivity (TFP). If these inputs are modeled as perfect substitutes but are in fact imperfect substitutes, then movements in the relative prices of these goods can impact measured TFP, either because they alter the number of varieties used or because they alter the quality mix of domestic and foreign inputs. Gopinath and Neiman (2011) study the Argentinean economy and find that such trade adjustments deliver quantitatively important declines in manufacturing TFP. In this paper we maintain the assumption that abstracting from heterogeneous inputs is a reasonable approximation for the U.S. productive sector as a whole.

GDP, $b_{F,t} \equiv B_{F,t}/\overline{Y}_t$, which evolves according to a first-order autoregressive process:

$$b_{F,t+1} = (1 - \rho_F)\,\overline{b} + \rho_F b_{F,t} + \sigma_F \eta_{t+1}, \tag{1}$$

where η_{t+1} has been normalized to have standard deviation equal to unity. The stochastic process (1) implies that external leverage relative to trend GDP reverts to a mean, $\overline{b}$. Thus, while some amount of the nation's debt is expected to be refinanced in perpetuity, amounts above the mean (such as those represented in recent data) are expected to be paid back rather than refinanced.

This process is calibrated to historical data on foreign holdings of U.S. Treasury debt (available from the Department of Treasury, U.S. Government). A grid for the state variable $b_{F,t}$ is used in the numerical solution. Both the grid span and the parameters of the AR(1) process for $b_{F,t+1}$ (1) are calibrated from historical data on foreign holdings of U.S. Treasury debt spanning the period 1984 to 2010. Estimation of the AR(1) process on these data produces values for $\rho_b = 0.95$, $\overline{b} = 0.148$, and $\sigma_b = 0.017$. The Appendix provides additional details on how this process is estimated from data.

An important feature of the process as written above is that the shocks η_{t+1} are exogenous, unrelated to the other primitive aggregate shocks in the model economy, namely two productivity shocks. Of course, foreign capital flows in the model will still be contemporaneously correlated with aggregate quantities and prices, since flows endogenously influence these variables in equilibrium. But there is no implication from the specification (1) that FOI holdings of the safe asset respond to the domestic economy.[11]

To investigate whether this specification is reasonable, we run Granger causality regressions of log changes in $b_{F,t+1}$ ("flows") on lagged log changes in GDP and lagged log change in two different measures of total factor productivity (TFP) from Fernald (2009). Table 1 presents results for 4-quarter log changes in these variables. Thus, we regress flows $\ln(b_{F,t}) - \ln(b_{F,t-4})$ on a constant, two lags of itself, two lags of the log difference in TFP, and two lags of the log difference in GDP.

The key observation from Table 1 is that foreign purchases of U.S. safe assets are essentially "explained" by lagged flows, not by lagged GDP growth or lagged TFP growth.

[11] Put differently, a potential concern is that positive productivity shocks cause a trade deficit and capital inflow, which we would attribute to a capital inflow shock rather than a productivity shock.

Lagged GDP growth and lagged TFP growth are statistically and economically insignificant explanatory variables. The fourth column shows that lagged GDP growth by itself explains just 3.7% of the variation in the log change in flows. This should be contrasted with the result in column 1, which shows that adding lagged flows allows the regression to explain 18% of the variation in log change in flows. To form a basis for comparison, column 5 shows that lagged GDP growth is a strong predictor of GDP growth itself, despite the finding that these lags explain virtually none of the movement in foreign purchases of U.S. safe assets.

We conclude that modeling changes in FOI purchases of safe assets as independent of the domestic economy is a reasonable first approximation. Note that this evidence and modeling approach does *not* imply or presume that the entire current account is exogenous. It applies only to flows on safe assets, which are dominated by FOI purchases. What the evidence above suggests is that this component is plausibly exogenous to the domestic economy. It is this component that we study here.

3 The Model

This section describes the model economy with two productive sectors. Time is discrete and each period t corresponds to a year. The economy grows deterministically at rate g. The exogenous aggregate shocks of the model include a stationary shock to foreign capital relative to trend GDP, and stationary technology shocks $Z_{k,t}$, one to each of the two sectors indexed by k, that have both a deterministic component and stochastic component, i.e., $Z_{k,t} = \exp(gt)\, z_{k,t}$, where $z_{k,t}$ is a stationary technology shock. The variable $\exp(gt)$ is trend output, interchangeably denoted $\overline{Y}_t \equiv \exp(gt)$.

3.1 Firms

The production side of the economy consists of two sectors, one producing a non-housing consumption good, and the other producing a housing good. We refer to the first as the "consumption sector" and the second as the "housing sector."

Denote output in the consumption sector as

$$Y_{C,t} \equiv Z_{C,t}^{1-\alpha} K_{C,t}^{\alpha} N_{C,t}^{1-\alpha}$$

where $Z_{C,t}$ is the stochastic productivity level at time t, K_C is the capital stock in the consumption sector, and N_C is the quantity of labor input in the consumption sector. Let I_C denote investment in the consumption sector. The firm's capital stock $K_{C,t}$ accumulates over time subject to proportional quadratic adjustment costs given by the function $\varphi\left(\frac{I_{C,t}}{K_{C,t}}-\delta\right)^2 K_{C,t}$, modeled as a deduction from the earnings of the firm. The firm does not issue new shares and finances its capital stock entirely through retained earnings. The dividends to shareholders are equal to

$$D_{C,t} = Y_{C,t} - \mathcal{W}_t N_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 K_{C,t}.$$

The firm maximizes the present discounted value $V_{C,t}$ of a stream of dividends:

$$V_{C,t} = \max_{N_{C,t}, I_{C,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{C,t}, \tag{2}$$

where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is a stochastic discount factor discussed in the appendix, and $\mathcal{W}_t$ is the wage rate (equal across sectors in equilibrium). The evolution equation for the firm's capital stock is

$$K_{C,t+1} = (1-\delta) K_{C,t} + I_{C,t},$$

where δ is the depreciation rate of the capital stock.

The housing firm's problem is analogous, except that housing production utilizes an additional fixed factor of production, $\mathcal{L}_t$, representing a combination of land and government permits for residential construction.[12] Denote output in the residential housing sector as

$$Y_{H,t} = (Z_{H,t}\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} Z_{H,t}^{1-\nu} N_{H,t}^{1-\nu}\right)^{\phi},$$

$Y_{H,t}$ represents construction of new housing (residential investment), $1-\phi$ is the share of land/permits in housing production, and ν is the share of capital in the construction

[12] Glaeser, Gyourko, and Saks (2005) argue that the increasing value of land for residential development is tied to government-issued construction permits, rather than to the acreage itself. We do not distinguish between these two forms of productive input and instead aggregate both forms into a single factor $\mathcal{L}_t$.

component $\left(K_{H,t}^{\nu} Z_{H,t}^{1-\nu} N_{H,t}^{1-\nu}\right)$ of housing production. Variables denoted with an "H" subscript are defined as above for the consumption sector, e.g., $Z_{H,t}$ denotes the stochastic productivity level in the housing sector.

Let p_t^H denote the relative price of housing in units of the non-housing consumption good, and let p_t^L denote the price of land/permits. The variable p_t^H is the time t price of a unit of housing of fixed quality and quantity; it corresponds to the value of a national house-price index. The dividends to shareholders in the housing sector are given by

$$D_{H,t} = p_t^H Y_{H,t} - p_t^L \mathcal{L}_t - \mathcal{W}_t N_{H,t} - I_{H,t} - \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 K_{H,t}.$$

The housing firm maximizes

$$V_{H,t} = \max_{N_{H,t}, I_{H,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{H,t}. \tag{3}$$

Capital in the housing sector evolves:

$$K_{H,t+1} = (1 - \delta) K_{H,t} + I_{H,t}.$$

Note that $Y_{H,t}$ represents residential investment; thus the law of motion for the aggregate residential housing stock H_t is

$$H_{t+1} = (1 - \delta_H) H_t + Y_{H,t},$$

where δ_H denotes the depreciation rate of the housing stock.

We assume that, each period, the government makes available a fixed supply $\bar{\mathcal{L}}$ of land/permits for residential construction by renting them at the competitive rental rate equal to the marginal product of $\mathcal{L}_t$. As described below, the proceeds from land/permits along with lump sum taxes are used to finance a (nonstochastic) amount of government borrowing in the risk-free bond market. When a house is sold, the government issues a transferable lease for the land/permits in perpetuity at no charge to the homeowner. Thus, the buyer of the home operates as owner even though, by eminent domain, the government retains the legal right to the land/permits.

3.2 Government Borrowing

We allow foreign inflows to finance government borrowing, as well as private borrowing. By doing so, high borrowing by current generations can influence unborn generations through future taxes and transfers. To keep the model tractable, we introduce non-stochastic government debt issuance in the risk-free bond market as follows. Let B_t^G be the government's *demand* for bonds, which we set to be a fixed fraction of trend GDP $B_t^G = b^G \overline{Y}_t$. In this calibration, B_t^G is negative since the government supplies bonds, hence is a net borrower. The parameter $b^G < 0$ is set to equal the (negative of) the observed ratio of government debt to trend GDP over the period 1984-2008.

At time $t+1$ the government raises funds by issuing new debt (negative bond demand) $-B_{t+1}^G = -b^G \overline{Y}_t \exp(g)$ at price q_t. Suppose additionally that the government can pay lump sum transfers T_t. Then the government's per period budget constraint implies that revenues from land/permits, revenues from new debt issuance, and revenues from negative transfers (lump sum taxes) must equal debt to be paid back this period:

$$
\begin{aligned}
p_t^L \mathcal{L}_t - B_{t+1}^G q_t - T_t &= -B_t^G \\
b^G \overline{Y}_t \left(\exp(g) q_t - 1\right) - p_t^L \mathcal{L}_t &= -T_t.
\end{aligned}
$$

Lump sum taxes equal the difference between the government's interest payments and land revenue. This insures that the interest payments are funded. We assume that T_t is distributed lump-sum across the population and denote the proportion of government lump sum transfers paid to individual i as T_t^i.

3.3 Individuals

The economy is populated by A overlapping generations of individuals, indexed by $a = 1, ..., A$, with a continuum of individuals born each period. Individuals live through two stages of life, a working stage and a retirement stage. Adult age begins at age 21, so a equals this effective age minus 20. Agents live for a maximum of $A = 80$ (100 years). Workers live from age 21 ($a = 1$) to 65 ($a = 45$) and then retire. Retired workers die with an age-dependent probability calibrated from life expectancy data. The probability that an agent is alive at age $a+1$ conditional on being alive at age a is denoted $\pi_{a+1|a}$.

Individuals have an intraperiod utility function given by

$$U(C_{a,t}^i, H_{a,t}^i) = \frac{\widetilde{C}_{a,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \widetilde{C}_{a,t} = \left(C_{a,t}^i\right)^{\chi} \left(H_{a,t}^i\right)^{1-\chi},$$

where $\widetilde{C}$ is referred to as *composite* consumption, $C_{a,t}$ is non-housing consumption of an individual of age a, and $H_{a,t}$ is the stock of housing, $1/\sigma$ is the coefficient of relative risk aversion, χ is the relative weight on non-housing consumption in utility. Implicit in this specification is the assumption that the service flow from houses is proportional to the stock $H_{a,t}$.

Individuals are heterogeneous in their labor productivity. To denote this heterogeneity, we index individuals i. Before retirement households supply labor inelastically. The stochastic process for individual income for workers is the product of $\mathcal{W}_t$, the aggregate wage per unit of productivity, and $L_{a,t}^i$, the individual's labor endowment (hours times an individual-specific productivity factor). Labor productivity is specified by a deterministic age-specific profile, G_a, and an individual shock Z_t^i:

$$\begin{aligned} L_{a,t}^i &= G_a Z_t^i \\ \ln\left(Z_t^i\right) &= \ln\left(Z_{t-1}^i\right) + \epsilon_t^i, \qquad \epsilon_t^i \sim i.i.d.\left(0, \sigma_t^2\right), \end{aligned}$$

where G_a is a deterministic function of age capturing a hump-shaped profile in life-cycle earnings and ϵ_t^i is a stochastic i.i.d. shock to individual earnings. To capture countercyclical variation in idiosyncratic risk of the type documented by Storesletten, Telmer, and Yaron (2004), we use a two-state specification for the variance of idiosyncratic earnings shocks:

$$\sigma_t^2 = \begin{cases} \sigma_E^2 & \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ \sigma_R^2 & \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \end{cases}, \qquad \sigma_R^2 > \sigma_E^2 \tag{4}$$

This specification implies that the variance of idiosyncratic labor earnings is higher in "recessions" ($Z_{C,t} \leq E\left(Z_{C,t}\right)$) than in "expansions" ($Z_{C,t} \geq E\left(Z_{C,t}\right)$). The former is denoted with an "R" subscript, the latter with an "E" subscript. Labor earnings are taxed at rate τ in order to finance social security retirement income. Upon death, any remaining net worth of an individual is transferred to a newborn who replaces her, via an accidental bequest.[13] Other than these accidental bequests, agents are not endowed with any risky assets or bonds at birth.

Financial market trade is limited to a one-period riskless bond and to risky capital, where the latter is restricted to be a mutual fund of equity in the housing and consumption sectors. The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where q_{t-1} is the bond price known at time $t-1$. At age a, agents enter the period with wealth invested in bonds, B_a^i, and shares θ_a^i of risky capital. The total number of shares outstanding of the risky asset is normalized to unity. Define the individual's gross financial wealth at time t as

$$W_{a,t}^i \equiv \theta_{a,t}^i \left(V_{C,t} + V_{H,t}\right) + B_{a,t}^i.$$

We rule out short-sales in the risky asset, $\theta_{a,t}^i \geq 0$,and assume that an agent who chooses to invest in the mutual fund pays a fixed, per-period participation cost, $F_{K,t}$.

We assume that the housing owned by each individual requires maintenance expenses $p_t^H H_{a,t}^i \delta_H$, where δ_H is the rate of depreciation of the aggregate housing stock. At time t, households may choose to change the quantity of housing consumed at time $t+1$ by selling their current house for $p_t^H H_{a,t}^i$ and buying a new house for $p_t^H H_{a,t+1}^i$. An individual who chooses to change housing consumption pays a transaction cost $F_{H,t}^i$, which contains both a fixed and variable component proportional to the value of the house.

One component of the transactions cost in illiquid housing is the cost directly associated with housing finance, specifically borrowing costs. We use direct evidence to calibrate a transactions cost, λ, per dollar borrowed, given by $F_{B,t}^i = \lambda |B_{a+1,t+1}^i|$, whenever $B_{a+1,t+1}^i < 0$, which represents a borrowing position in the risk-free asset. Denote the sum of these costs for individual i as $F_t^i \equiv F_{K,t} + F_{H,t}^i + F_{B,t}$, where

[13] The collateral constraint (Equation 6 below) implies that net worth is non-negative so that accidental bequests are non-negative. If household i dies in period t then his net worth at death, left accidentally, is equivalent to the amount inherited by the newborn who replaces the dead individual. We allow the newborn to make an optimal portfolio choice over risky assets, bonds, and housing for how the bequeathed wealth is allocated in the first period of life. Favilukis, Ludvigson, and Van Nieuwerburgh (2008) study a model in which, in addition to these accidental bequests, some small fraction of households leave intentional bequests, driven by a bequest motive in their value functions.

$$F^i_{H,t} = \begin{cases} 0 & \text{if } H^i_{a+1,t+1} = H^i_{a,t} \\ \psi_0 + \psi_1 p^H_t H^i_{a,t} & \text{if } H^i_{a+1,t+1} \neq H^i_{a,t} \end{cases}.$$

$$F_{K,t} = \begin{cases} 0 & \text{if } \theta^i_{a+1,t+1} = 0 \\ \overline{F} & \text{if } \theta^i_{a+1,t+1} > 0 \end{cases}.$$

$$F^i_{B,t} = \begin{cases} 0 & \text{if } B^i_{a+1,t+1} > 0 \\ \lambda \cdot \left|B^i_{a+1,t+1}\right| & \text{if } B^i_{a+1,t+1} < 0 \end{cases}.$$

The budget constraint for an agent of age a who is not retired is

$$\begin{aligned} & C^i_{a,t} + p^H_t \delta_H H^i_{a,t} + B^i_{a+1,t+1} q_t + \theta^i_{a+1,t+1} \left(V_{C,t} - D_{C,t} + V_{H,t} - D_{H,t}\right) \\ \leq \; & W^i_{a,t} + (1-\tau)\,\mathcal{W}_t L^i_{a,t} + p^H_t \left(H^i_{a,t} - H^i_{a+1,t+1}\right) - F^i_t + T^i_t \end{aligned} \tag{5}$$

where τ is a social security tax rate. Equation (5) says that the amount spent on non-housing consumption, on housing maintenance, and on bond and equity purchases must be less than or equal to the sum of the individual's gross financial wealth and after-tax labor income, less the cost of purchasing any additional housing, less all asset market transactions costs.

A key constraint in the model is a collateral constraint taking the form

$$-B^i_{a+1,t+1} \leq (1-\varpi)\, p^H_t H^i_{a,t+1}, \qquad \forall a, t \tag{6}$$

where $0 \leq \varpi \leq 1$. The constraint says that households may borrow no more than a fraction $(1-\varpi)$ of the value of housing, implying that they must post collateral equal to a fraction ϖ of the value of the house.

Each period, retired workers receive a government pension $PE^i_{a,t} = Z^i_{ar} X_t$, where $X_t = \mathcal{W}_t \tau \left(\frac{N^W}{N^R}\right)$ is the pension determined by a pay as you go system, Z^i_{ar} denotes the value of the stochastic component of individual labor productivity during the last year of working life, and N^W and N^R are the numbers of working age and retired households. For agents who have reached retirement age, the budget constraint is identical to that for workers (5) except that wage income $(1-\tau)\,\mathcal{W}_t L^i_{a,t}$ is replaced by pension income $PE^i_{a,t}$.

Let $Z_t \equiv (Z_{C,t}, Z_{H,t}, B_{F,t}, B_{F,t+1})'$ denote the exogenous aggregate states faced by an individual. The total aggregate state of the economy is a pair, (Z, μ), where μ is a measure defined over $\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathfrak{W} \times \mathcal{H})$, where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathfrak{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ is a distribution of agents across ages, idiosyncratic shocks, financial and housing wealth. The presence of aggregate shocks implies that μ evolves stochastically over time. We approximate μ numerically and specify a law of motion for it, as described in the appendix. We denote this law of motion Γ:

$$\mu_{t+1} = \Gamma\left(\mu_t, Z_t, Z_{t+1}\right).$$

This completes the description of the model economy. We now turn to the definition of equilibrium.

3.4 Equilibrium

An equilibrium is defined as a set of prices (bond prices, wages, risky asset returns, house price, and land price) given by time-invariant functions $q_t = q\left(\mu_t, Z_t\right)$, $\mathcal{W}_t = \mathcal{W}\left(\mu_t, Z_t\right)$, $R_{K,t} = R_K\left(\mu_t, Z_t\right)$, $p_t^H = p^H\left(\mu_t, Z_t\right)$, and $p_t^L = p_t^L\left(\mu_t, Z_t\right)$, respectively, a set of cohort-specific value functions and decision rules for each individual i, $\left\{\upsilon_a, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}\right\}_{a=1}^{A}$ and a law of motion for μ, $\mu_{t+1} = \Gamma\left(\mu_t, Z_t, Z_{t+1}\right)$ such that households and firms optimize, the aggregate law of motion of the economy is consistent with individual behavior, and all markets clear. The appendix contains all equilibrium conditions. Here we single out the equilibrium condition that pins down bond prices $q_t = q\left(\mu_t, Z_t\right)$ such that the demand for U.S. government bonds from domestic agents and from abroad equals the supply:

$$\int_{\mathcal{S}} B^i_{a,t} d\mu + B_{F,t} + B_{G,t} = 0. \tag{7}$$

Define aggregate quantities C_t and F_t as

$$C_t \equiv \int_{\mathcal{S}} C^i_{a,t} d\mu \qquad F_t \equiv F_{K,t} + F_{B,t} + \int_{\mathcal{S}} F^i_{H,t} d\mu. \tag{8}$$

The aggregate resource constraint for the economy must take into account the housing and risky capital market transactions/participation costs, which reduce consumption, the adjustment costs in productive capital, which reduce firm profits, and the change in net foreign capital in the bond market, which finances domestic consumption and investment. Thus, non-housing output equals non-housing consumption (inclusive of total financial transactions costs F_t) plus aggregate investment (gross of adjustment costs) less the change in the value of net foreign holdings:

$$Y_{C,t} = C_t + F_t + \left(I_{C,t} + \varphi \left(\frac{I_{C,t}}{K_{C,t}} - \delta \right)^2 K_{C,t} \right) + \left(I_{H,t} + \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 K_{H,t} \right) \quad (9)$$
$$\underbrace{- \left(B_{t+1}^F q\left(\mu_t, Z_t\right) - B_t^F \right)}_{\text{trade balance}},$$

where the term labeled "trade balance" is equal to the current account plus net financial income from abroad, i.e., current account = trade balance $- \left(1 - q\left(\mu_t, Z_t\right)\right) B_t^F$.[14] Alternatively, current account = minus the change in the value of net foreign holdings of domestic assets = $- \left(B_{t+1}^F - B_t^F \right) q\left(\mu_t, Z_t\right)$.

To solve the model, it is necessary to approximate the infinite dimensional object μ with a finite dimensional object. The appendix explains the solution procedure and how we specify a finite dimensional vector to represent the law of motion for μ. The resulting approximation, or "bounded rationality" equilibrium has been used extensively in the literature to solve incomplete markets models (see the appendix for further discussion).

3.5 Welfare Measure

To quantify the welfare effects of different foreign holdings regimes, we use a consumption equivalent variation measure. To explain this measure, it is necessary to introduce some additional notation. Let H_t without an i subscript denote aggregate housing wealth, i.e., $H_t \equiv \int_{\mathcal{S}} H_{a,t}^i d\mu$, and analogously for other individual variables. To study a growing economy, it will be convenient to normalize trending variables by trend output and denote their deterministically detrended values in lower case, e.g., $z_{c,t} \equiv Z_{c,t} \exp\left(-gt\right)$, $h_t^i \equiv H_t^i \exp\left(-gt\right)$, etc. The solved policy functions and state variables are expressed in terms of normalized variables.

[14] Note that (9) simply results from aggregating the budget constraints across all households, imposing all market clearing conditions, and using the definitions of dividends as equal to firm revenue minus costs.

As explained in the appendix, the bounded rationality equilibrium is computed by approximating the infinite dimensional object (Z_t, μ_t) with a finite dimensional vector of aggregate state variables given next. Let the subset of aggregate state variables *excluding* foreign bonds be approximated by μ_t^{AG}:

$$(z_{C,t}, z_{H,t}, \mu_t) \approx \mu_t^{AG} = \left(z_{C,t}, z_{H,t}, k_{C,t}, \frac{k_{C,t}}{k_{C,t} + k_{H,t}}, h_t, p_t^H, q_t \right).$$

We may write the household value function as a function of detrended variables as

$$v_a(\mu_t^{AG}, b_{F,t}, b_{F,t+1}, Z_t^i, w_t^i, h_t^i).$$

Integrating out aggregate risk except foreign bonds we have

$$\bar{v}_a(b_{F,t}, b_{F,t+1}, Z_t^i, w_t^i, h_t^i) = \int v_a(\mu_t^{AG}, b_{F,t}, b_{F,t+1}, Z_t^i, w_t^i, h_t^i) f_{\mu^{AG}}\left(\mu_t^{AG}\right) d\mu_t^{AG},$$

where $f_{\mu^{AG}}\left(\mu_t^{AG}\right)$ is the probability density function of μ_t^{AG}.

We quantify the welfare consequences of different foreign capital states by computing the increment to lifetime utility (the household value function) in units of the composite (housing plus nonhousing) consumption good, of being in a high versus low state of foreign capital holdings relative to trend GDP. We call this a consumption "equivalent variation" (EV) measure. For example, we can compute the equivalent variation measure for individual i of age a that would result from transitioning into a different foreign capital state at $t+1$ by an increment Δ, compared to remaining in a particular foreign capital state $b_{F,t+1} = b_{F,t}$:

$$EV_{i,a} = \left(\frac{\bar{v}_a(b_{F,t}, b_{F,t} + \Delta, Z_t^i, w_t^i, h_t^i)}{\bar{v}_a(b_{F,t}, b_{F,t}, Z_t^i, w_t^i, h_t^i)} \right)^{\frac{\sigma}{\sigma-1}} - 1. \tag{10}$$

The equivalent variation measure tells us how much this individual's lifetime composite consumption must be increased so that her utility from remaining in a particular foreign capital state $b_{F,t}$ equals that from transitioning to $b_{F,t} + \Delta$. (We multiply the units by 100 so as to express them in percent.) Positive numbers therefore reflect a welfare gain from transitioning, whereas negative numbers reflect a welfare loss.

We use a similar criterion to compute an ex-ante welfare measure under the "veil of ignorance." That is, we compute the welfare implications of a change in foreign holdings for an agent about to be born (age = 0) with the average idiosyncratic productivity, $Z_t^i = 1$, whose financial wealth, $W^i_{0,t}$, and housing wealth, $H^i_{0,t}$, are optimally chosen prior to entering the model based on the accidental bequest inherited from the dead. This is computed using that agent's value function at the start of life, which incorporates the agent's expectation of lifetime utility over all possible aggregate and idiosyncratic shocks in the future, i.e.,

$$EV_{NB} = \left(\frac{\bar{v}_1(b_{F,t}, b_{F,t} + \Delta, 1, 0, h_0)}{\bar{v}_1(b_{F,t}, b_{F,t}, 1, 0, h_0)} \right)^{\frac{\sigma}{\sigma-1}} - 1. \tag{11}$$

Finally, we compare the welfare consequences for more aggregated demographic groups in a similar manner, averaging EV across such groups.

The integrals are computed as averages from a very long simulated sample path. We locate all dates in this path for which $b_{F,t}$ is equal to a particular value $\widetilde{b}$, and for which $b_{F,t+1}$ is equal to $\widetilde{b} + \Delta$, and then locate all dates in which $b_{F,t} = b_{F,t+1} = \widetilde{b}$.[15] We then form the ratio $\bar{v}_a(\widetilde{b}, \widetilde{b} + \Delta, Z^i_t, w^i_t, h^i_t)/\bar{v}_a(\widetilde{b}, \widetilde{b}, Z^i_t, w^i_t, h^i_t)$ and average this ratio over the relevant subgroup of the population. We set the increment, Δ, equal to a typical increase or decrease in foreign holdings given the stochastic process (1), i.e., $\Delta = (1 - \rho_F)\,\bar{b} + \rho_F b_{F,t} + \sigma_F \cdot 1$ (increase) or $\Delta = (1 - \rho_F)\,\bar{b} + \rho_F b_{F,t} + \sigma_F \cdot (-1)$ (decrease).

3.6 Model Calibration

The numerical calibration of the model's parameters are reported in Table 2. A detailed explanation of this calibration, including individual and aggregate productivity shocks, is given in the Appendix. The technology shocks Z_C and Z_H are assumed to follow two-state independent Markov chains. Because most of the parameter calibrations are either standard or follow from previous papers, we provide the discussion over these values in the Appendix.

4 Benchmark Results

This section presents the model's main implications. Unless otherwise noted, these implications are based on long simulations of the model using the solved optimal policy functions and evolution equations for the state variables. Before turning to the welfare implications of

[15] In practice, this is accomplished by locating all points within a close radius of a particular value.

changes in foreign holdings, we present the model's predictions for a set of benchmark business cycle and asset pricing statistics, and we study how these statistics depend on foreign capital flows into U.S. safe assets.

Table 3 presents benchmark results for HP-detrended aggregate quantities. We report statistics for total output, $GDP \equiv Y = Y_C + p^H Y_H + C_H$, non-housing consumption (inclusive of expenditures on financial services), equal to $C_t + F_t$, housing consumption $C_{H,t}$, defined as price per unit of housing services times quantity of housing or $C_{H,t} \equiv \mathcal{R}_t H_t$, total (housing and non-housing) consumption $C_{T,t} = C_t + F_t + C_{H,t}$, non-housing investment (inclusive of adjustment costs) $I_t = (I_{C,t} + \phi_C(\cdot) K_{C,t}) + (I_{H,t} + \phi_H(\cdot) K_{H,t})$, residential investment, $p_t^H Y_{H,t}$, and total investment $I_{T,t} = I_t + p_t^H Y_{H,t}$.

The standard deviation of total aggregate consumption divided by the standard deviation of GDP is 0.63 in the model, identical to the 0.63 value found in the data. In addition, the level of GDP volatility in the model is close to that in the data. The model produces a plausible amount of aggregate consumption volatility. Total investment is more volatile than output, both in the model and in the data, and the model produces about the right amount of relative volatility: the ratio of the standard deviation of total investment to that of GDP is 3.57 in the model compared to 2.95 in the data. The model does a good job of matching the relative volatility of residential investment to output: in the data the ratio of these volatilities is 4.65, while it is 5.14 in the model. Finally, both in the model and the data, residential investment is less correlated with output than is consumption and total investment. But the model somewhat understates the share of consumption in GDP.

To get a sense of the how aggregate business cycle statistics are affected by the quantities of foreign holdings of domestic assets, as well as by a capital inflow (outflow), Table 4 presents the mean and standard deviation of the (detrended) aggregate variables, conditional on the stock of foreign holdings as of last period, $b_{F,t}$ (external leverage), as well as on the change (flow) in foreign holdings this period, $\Delta b_{F,t+1}$. The statistics are reported conditional on being in the top or bottom half of the sample in terms of these variables, distinguished as high values, "H" or low values, "L." In computing these statistics, we average out over the other aggregate shocks in the economy (the productivity shocks) using long simulations, thereby isolating the effect of external leverage on the economy.

A capital inflow, which represents a negative trade balance or a current account deficit, finances domestic spending and therefore acts like a positive economic shock. Table 4 shows that a high capital inflow stimulates investment and consumption: the means of these variables are higher when capital inflows are high than when inflows are low. Total investment is 15% higher (mean of 0.7 versus 0.61) in high inflow states than in low inflow states, while residential investment is 25% higher. This leads GDP to be about 1% higher. A high capital inflow also makes consumption and investment less volatile, relative to low capital inflow states. Investment volatility falls because an inflow leads to a higher level of investment. With convex adjustment costs, the volatility of investment is reduced because the cost of any given change in investment is higher when the level of investment is high.

A high *stock* of external leverage, $b_{F,t}$, has a smaller impact on consumption and investment. This happens because, although some of the debt relative to trend GDP is expected to be rolled over indefinitely, amounts above average are not. Thus, the stimulatory affect of past inflows is dampened by the expectation that some of the debt must eventually be repaid, as capital flows relative to GDP slowly mean-revert.

Table 5 shows the sensitivity of the log difference in aggregate variables to both the level of external holdings last period, $b_{F,t}$, and the capital flow $\Delta b_{F,t}$, from a multivariate regression on these variables. A capital inflow $\Delta b_{F,t+1}$ stimulates higher economic growth (consumption, investment and GDP), and higher growth in the capital stocks at the beginning of next period (housing and physical capital). The marginal effect of a high capital inflow also raises the real wage (row 10), since the influx of foreign funds stimulates growth in the capital stock and, along with it, the value of the relatively scarce factor (labor). Controlling for the stimulatory impact of a higher capital inflow, however, a high level of external debt has a small contractionary effect on spending, since above mean levels of debt relative to trend GDP must eventually be repaid. The effects of these changes in the stock of external debt on investment are too small to have a discernible influence on the slow-moving physical and housing capital stocks.

Table 6 reports the model's implications for asset pricing moments. The table reports unconditional moments in the "all" column, as well as moments conditional on either the stock of foreign holdings last period, $b_{F,t}$, or on the flow this period, $\Delta b_{F,t+1}$, being above or below average. The Sharpe ratio for each asset, denoted $SR[\cdot]$, is defined to be the mean of the return on the asset in excess of the risk-free rate, divided by the standard deviation of this excess return.

The benchmark model comes close to matching the historical mean return for the risk-free rate but somewhat overstates the volatility of the risk-free rate. The model produces a sizable equity return of 6.7% per annum, an annual equity premium of 6.1%, and annual Sharpe ratio of 0.67. Two factors related to the cyclicality of the cross-sectional distribution of consumption contribute to the model's high average risk premium and Sharpe ratio. First, idiosyncratic income risk is countercyclical. Second, house prices and therefore collateral values are procyclical, making borrowing constraints countercyclical. These factors mean that insurance/risk-sharing opportunities are reduced precisely when households need them most (in recessions). The model produces about the right mean return for the aggregate house price index. The mean housing return is 8.06% on an annual basis, with Sharpe ratio equal to 1.33, comparable to U.S. data for aggregate house price indexes.[16]

One shortcoming of the present setup is that the volatility of the equity return is about 60% of what it is in post-war data, reflecting a well known trade-off in production-based models with adjustment costs between matching the volatility of investment and the volatility of equity returns. One potential resolution is to increase adjustment costs while at the same time introducing additional shocks to offset the reduction in the volatility of investment, or investment-specific technology shocks. We do not pursue these possibilities here in order to keep the complexity of the model to a minimum and the numerical solution procedure manageable.

The right-most four columns of Table 6 show asset pricing moments conditional on the amount of external leverage, $b_{F,t}$, or conditional on high or low capital inflows, $\Delta b_{F,t+1}$. Both a high level of external leverage and a high capital inflow lead to a sharp decline in the riskless interest rate and in the expected return on equity. At the same time, however, an inflow leads to an increase in the equity risk premium and Sharpe ratio. The Sharpe ratio on equity

[16] The housing Sharpe ratio in the model pertains to an *aggregate* house index return. Individual houses are subject to significant idiosyncratic risk that is averaged out in the aggregate index and would have much lower Sharpe ratios. See the appendix for additional discussion.

is 13% higher in high capital inflow states than in low capital inflow states. Risk premia rise because the inflow reduces the effective supply of the safe asset, forcing domestic savers to hold more of their funds in the form of risky securities. Thus, although total expected returns (discount rates) fall in response to a capital inflow, the risk-premium *component* of the discount rate rises.

Results are similar for the housing asset. A high level of external leverage leads to a higher housing risk premium, as does an inflow. The rise in risk premia in turn partially (but not fully) offsets the stimulatory impact of a lower riskless interest rate on home prices, partly explaining why there is only a modest increase in the price-rent ratio in response to a high level of external leverage (row 13). (The response of aggregate homes to a foreign inflow is also limited by the equilibrium increase in residential investment, as discussed below.)

In contrast to the modest impact a foreign capital inflow has on the price-rent ratio, the stock price-dividend ratio (V/D) responds sharply to a capital inflow and is 50% higher than average, conditional on a capital inflow. This occurs because a capital inflow is met with a significant increase in expected dividend growth that is not present for expected rent growth. Indeed, positive economic shocks, which stimulate residential investment, are associated with an expectation of *lower*, rather than higher, future rental growth, an effect that, but for the offsetting decline in discount rates, would *reduce* the price-rent ratio.[17] Taken together, these elements of the model imply that a reserve-driven capital inflow of the type that occurred in the last 15 years can have, at most, a limited impact on home prices.[18]

Table 6 (columns 4 and 5) also shows that a high level of external leverage, $b_{F,t}$, raises both the risk premium on equity and housing and the volatility of these assets, as domestic households on the whole are now in a more levered portfolio position.[19]

[17] Rents are inversely related to the housing stock because the implicit rental price for housing services is positively related the marginal utility of housing services relative to the marginal utility of non-housing services. By contrast, expected future profits of the productive sector are positively related to an expansion of the physical capital stock because the resulting increase in the marginal product of labor more than offsets the marginal cost of new investment.

[18] It follows that other factors must be primarily responsible for the large boom-bust cycle in home prices that occurred from 2000-2010. Favilukis, Ludvigson, and Van Nieuwerburgh (2008) argue that plausibly calibrated changes in collateral requirements and housing transactions cost can account for the run-up and subsequent decline in U.S. aggregate house price-rent ratios.

[19] This outcome is the same as that in Caballero and Krishnamurthy (2009), who study a two-asset (equity and risk-free rate) representative agent exchange economy in which foreign demand for the safe asset is

Finally, we investigate how international capital flows affect the growth rates of asset values, returning to the last two rows of Table 5. A capital inflow stimulates growth in the aggregate value of the risky mutual fund as well as in housing wealth, $p_t^H H_{t+1}$. But conditional on the inflow, the level of external finance depresses asset values as the financing burden of higher external debt takes its toll on domestic spending and ultimately on asset valuations.

The relationship between external leverage and risk premia in the model is worthy of emphasis. In equilibrium, both a capital inflow and a high *level* of external leverage $b_{F,t}$ *raise* risk premia on housing and equity, rather than lower them. This runs contrary to the argument, made by some, that the free flow of capital across borders should be associated with a reduction in risk premia (e.g., Geithner (2007)). Here, foreign purchases of the safe asset make both domestic equity and domestic housing assets *more risky*, both because a higher level of external leverage forces domestic residents as a whole to take a leveraged position in the risky assets, and because the quantity of systematic risk per unit volatility borne by domestic savers rises.

5 Welfare Implications

We now turn to the welfare effects of international capital flows into the risk-free asset. Figure 3 shows the EV measure integrated out across all values of $b_{F,t}$ (dashed lines), as well as conditional on the economy residing in particular quintiles of $b_{F,t}$ (solid lines). The consequences of a capital inflow are shown in the left panel and a capital outflow in the right panel.

Figure 3 shows that the welfare implications of a capital inflow or outflow are non-

perfectly correlated with (but less volatile than), domestic consumption. But unlike here, in their model a capital inflow (an *increase* in external leverage) lowers equity risk premia. The reasons for this difference are three-fold. First, capital flows in Caballero and Krishnamurthy (2009) are assumed to be more stable than domestic cash flows, which lowers risk premia by stabilizing the economy. Here, capital flows are independent of the aggregate state and have innovations that are about as volatile as GDP. Second, unlike Caballero and Krishnamurthy (2009), international capital flows are not perfectly correlated with domestic cash flows; they therefore add systematic risk to the economy, uncorrelated with the aggregate risk already there. Third, Caballero and Krishnamurthy (2009) solve their model in continuous time, so that the instantaneous effect of a capital inflow has a negligible effect on leverage. By contrast, in the discrete time setting here, capital inflows have an immediate effect on external leverage.

monotone in age, and that the effects are potentially sizable. An increase in foreign holdings ($\Delta > 0$) benefits the young (age 35 or less), while a decrease is costly. The young benefit from a capital inflow due to lower interest rates, which reduce the costs of home ownership and of borrowing in anticipation of higher expected future income attributable to the hump-shaped life-cycle profile in earnings. An inflow stimulates the real wage (Table 5), which also benefits the young, a group with many years of working life ahead. At the average level of external leverage, $b_{F,t}$, the youngest households require about 0.20% more lifetime consumption to make them as well off as they would be from transitioning to a state where external leverage is higher for one year by the typical annual increment Δ. This value is slightly lower than when external leverage is at the lowest quintiles of the distribution. Conversely, a capital outflow hurts the young. The average EV measure associated with a decline in foreign holdings over all quintiles is -0.20% for the youngest households.

The welfare consequences are reversed for middle-aged households (age 45 to 72), who are significantly hurt by a capital inflow when the level of external leverage is sufficiently low. This occurs despite the fact that, like the young, many middle-aged households of working age still benefit from higher wages. The reason is that they are crowded out of the safe bond market and exposed to greater systematic risk in equity markets. Although they are partially compensated for this in equilibrium by higher risk-premia, they still suffer from lower expected rates of return on all assets, including the riskless bond, equity, and housing. The net effect is that middle-aged savers experience a welfare loss from an inflow and, conversely, a welfare gain from a capital outflow.

Two points about this result bear noting. First, the absolute value of the EV measure is about the same as that for the youngest households, but of the opposite sign, indicating the middle-aged households benefit from an outflow by about as much as the youngest households are hurt. But this is not true for an increase in foreign holdings ($\Delta > 0$): the EV welfare measure for 65 year-old households in the lowest foreign holdings quintile is -0.5%, twice as large in absolute value as the gain for young households in that quintile. Second, the EV measure for sixty five year-olds associated with a decrease in foreign holdings is negative, conditional on being in the highest foreign leverage quintile, implying that middle-aged households benefit from an outflow only when the level of external leverage is sufficiently low.

Results not reported show that this latter result can be explained by the behavior of expected asset returns, which respond more to a capital flow when the level of external leverage is low compared to when it is high. Thus, for example, an *out*flow causes a larger *increase* in expected returns when the level of external leverage is low compared to when it is high. Higher expected returns are beneficial for middle-aged savers, but they can only improve overall welfare if they rise by enough to offset the negative welfare consequences from both lower wages and lower asset values that also accompany an outflow.

We now consider the welfare consequences for older households of an increase in foreign holdings. The left panel of Figure 3 shows that older retired individuals benefit significantly by an increase in foreign ownership of the safe asset. They gain from the increase in the value of their assets when there is an inflow. Because they are dissaving at the end of life, they are less hurt than are middle-aged savers by a decline in expected returns. And because retirees earn a pension that is in large part determined by their earnings in the last period of working life, they are less exposed to systematic risk than are individuals of working age, whose labor earnings vary not only with the current aggregate state but also with the countercyclical fluctuation in the variance of idiosyncratic earnings surprises–see (4).[20] Taken together, these factors imply that older households experience a significant net gain from even modest increases in asset values that accompany a capital inflow. The right panel of Figure 3 shows that, from the highest external leverage quintile, the oldest individuals would be willing to give up 1.4% of lifetime consumption in order to avoid transitioning to a state where external leverage is lower for one year by a typical annual increment. This effect could be several times larger for a greater-than-typical decline, and many times larger for a series of annual declines in succession or over the remainder of the household's lifetime.

Figure 4 decomposes the welfare costs of a capital *out*flow by age, income and wealth. Young individuals who are high-income and especially those who are wealthy suffer less from a capital outflow than low-income or poor households because they are better equipped to

[20] Pension (Social Security) income is not entirely insulated from aggregate risk, since the pay-as-you-go system depends on tax revenue, which in turn depends on the current wage. But it is still far less sensitive to the current aggregate state than is labor income.

self-insure against idiosyncratic and aggregate risks without the benefit of easier borrowing terms that foreign capital provides. By contrast, high net worth retirees suffer *more* than low net worth retirees, because they have the most to lose from an outflow, namely a decline in the value of their assets. A similar pattern occurs when comparing high versus low income retirees because income and net worth are positively correlated in the model.

As a final welfare computation, we calculate the costs of a capital *out*flow under the "veil of ignorance" using newborn equivalent variation measure EV_{NB} in (11). The measure compares the value function of a newborn, born into the fifth quintile of today's foreign holdings, $b_{F,t+1} = b_{5,t+1}$, with the value function of a newborn born into each of the other quintiles, $b_{F,t+1} = b_{1,t+1}$, $b_{2,t+1}$,..., $b_{4,t+1}$. Since (11) depends on last period's bond holdings $b_{F,t}$, we integrate EV_{NB} out against the distribution of previous-period bond holdings $b_{F,t}$. The measure thus compares the lifetime utility functions of two newborns starting working life with different levels of external leverage $b_{F,t+1}$ but with the same initial idiosyncratic productivity draw, equal to the average across agents and over time value of $Z_t^i = 1$. The newborn computation depends on the initial level of initial wealth bequeathed to the newborn. We therefore compute this measure at the median, 25th and 75th percentiles of the newborn wealth distribution. The measure summarizes the expected welfare effects of an increase in external leverage, over the life cycle, as experienced by a newborn whose stochastic path of future earnings and foreign capital flows is unknown.

Figure 5 shows that newborns born into high foreign capital states are better off than those born into low capital states as long as they start life with wealth in the 25th or 50th percentile of the newborn wealth distribution. The poorest newborns who come into a world where foreign holdings are in their highest quintile would be willing to give up 2.7% of lifetime consumption to avoid being born into a world where the foreign holdings distribution is in their the lowest quintile. The cumulative effects over the life-cycle can be non-trivial, especially when contemplating a large capital flow. By contrast, the wealthiest newborns actually benefit slightly from an outflow. From Figure 4 we see that the wealthiest are not hurt by an outflow until old age, and they are slightly helped during middle age. But the negative consequences of an outflow during old age are heavily discounted from the perspective of a newborn, since the average agent in the model lives to age 80 (a lifespan of 60 years) and the debt/GDP ratio is stationary and expected to mean revert over long horizons.

6 Conclusion

The last two decades have been marked by a steady rise in international ownership of U.S. assets considered to be safe stores-of-value. Some have argued that these trends are optimal or benign, and/or that countries like the United States ultimately benefit from easier borrowing terms (e.g., Dooley, Folkerts-Landau, and Garber (2005), Cooper (2007), Mendoza, Quadrini, and Rios-Rull (2009), Caballero, Fahri, and Gourinchas (2008a)). Others (sometimes at the same time) have warned of the hazards of ever-increasing external leverage, and of the greater systematic risk that accompanies it (Caballero, Fahri, and Gourinchas (2008a), Caballero, Fahri, and Gourinchas (2008b), Obstfeld and Rogoff (2009), Fahri, Gourinchas, and Rey (2011)). Missing from this analysis are equilibrium models of aggregate and idiosyncratic risks, plausible financial markets, and household heterogeneity with which to study the welfare consequences of these global capital flows.

In this paper, we turn to such an equilibrium theory in order to better our understanding of the distributional consequences of this brand of international capital flow. We find that these flows confer on U.S. households both costs and benefits, both greater risk and greater opportunities to insure against risk, and that they do so concurrently. The relevant question is, which households are privy to the benefits and which are subject to the costs?

The model we study implies that foreign governmental demand for U.S. safe assets *does* lead to easier borrowing terms in the U.S., which benefits young households who are in a borrowing stage of the life-cycle, unless they are relatively wealthy and less reliant on borrowed funds. It also especially benefits the wealthy old, both because they gain from the rise in asset values that accompanies a capital inflow, and because they have the least to lose from lower expected rates of return and from the greater exposure of domestic saving to systematic risk. On the other hand, such flows are potentially quite costly for the middle-aged, who find their retirement savings earning lower expected rates of return on portfolios increasingly tilted towards assets with greater systematic risk. This phenomenon is reflected in a sharp rise in the equilibrium risk premium on equity and housing assets, and in a decline in the lifetime utility of households saving for retirement. Our computations imply that the intergenerational tradeoffs in welfare can be sizable. For an individual just beginning working life, the consequences crucially depend on how much wealth she begins life with: a large sell-off by foreign governments of their holdings of U.S. safe assets would be expected (under the veil of ignorance) to be fairly costly for the poorest newborns but slightly beneficial for the richest.

It is commonly believed that a large influx of international capital (on the order of magnitude experienced in the U.S. over the last 15 years) played an important role in U.S. home price appreciation. If so, one might have expected middle-aged savers who own homes to gain, rather than lose, from a capital inflow. As the model shows, this reasoning ignores the general equilibrium response of both residential investment and risk-premia that accompany a capital inflow. A capital inflow acts like a positive economic shock, provoking a rise in residential investment and reducing the expected growth rate of the dividend that housing pays. At the same time, inflows into the safe asset cause an increase in the housing risk premium, implying that discount rates fall less than the decline in the risk-free rate alone. Both of these factors work to offset the stimulatory impact of a capital inflow-driven decline in interest rates on home prices, thereby limiting this potential source of welfare gains to middle-aged homeowners. Of course, these same factors limit house price appreciation for all homeowners, including older homeowners who ultimately benefit from the capital inflow. The difference is that older households don't bear as much systematic risk in their retirement earnings as workers do in their labor earnings, and they suffer far less than those still accumulating wealth for retirement from lower expected rates of return.

References

Aguiar, M., and M. Amador (2011): "Growth In The Shadow Of Expropriation," *Quarterly Journal Of Economics*, 126(2), 651–697.

Alfaro, L., S. Kalemli-Ozcan, and V. Volosovych (2011): "Soverigns, Upstream Capital Flows, and Global Imbalances," Unpublished paper, Harvard Business School.

Benninga, S., and A. Protopapadakis (1990): "Leverage, Time Preference and the Equity Premium Puzzle," *Journal of Monetary Economics*, 25, 49–58.

Bernanke, B. S. (2011): "International Capital Flows and the Returns to Safe Assets in the United States 2003-2007," *Banque de France Financial Stability Review*, 15, 13–26.

Caballero, R. J. (2009): "Discussion of "Global Imbalances and the Financial Crisis: Products of Common Causes, by M.Obstfeld and K.Rogoff"," Unpublished paper, M.I.T.

Caballero, R. J., E. Fahri, and P.-O. Gourinchas (2008a): "An Equilibrium Model of "Global Imbalances"and Low Interest Rates," *American Economic Review*, 98(1), 358–393.

——— (2008b): "Financial Crash, Commodity Prices, and Global Imbalances," *Brookings Papers on Economic Activity*, pp. 1–55.

Caballero, R. J., and A. Krishnamurthy (2009): "Global Imbalances and Financial Fragility," *American Economic Review Papers and Proceedings*, 99, 584–588.

Calvo, G. A., A. Izquierdo, and E. Talvi (2006): "Sudden Stops and Phoenix Miracles in Emerging Markets," *American Economic Review*, 96(2), 405–410.

Cooper, R. N. (2007): "Living With Global Imbalances," *Brookings Papers on Economic Activity*, 2, 91–107.

Davis, M. A., and J. Heathcote (2005): "Housing and the Business Cycle," *International Economic Review*, 46(3), 751–784.

Dooley, M., D. Folkerts-Landau, and P. Garber (2005): *International Financial Stability: Asia, Interest Rates, and the Dollar*. Deutsche Bank Global, New York.

Fahri, E., P. O. Gourinchas, and H. Rey (2011): "Reforming the International Monetary System," Unpublished paper, Harvard University.

Favilukis, J. (2013): "Wealth Inequality, Stock Market Participation, and the Equity Premium," *Journal of Financial Economics*, 107(3), 740–759.

Favilukis, J., D. Kohn, S. C. Ludvigson, and S. Van Nieuwerburgh (2013): "International Capital Flows and House Prices: Theory and Evidence," in *Housing and the Financial Crisis*, ed. by E. L. Glaeser, and T. Sinai, pp. 235–299. University of Chicago Press, Chicago, IL.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2008): "The Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium," Unpublished paper, New York University.

Fernald, J. (2009): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," Unpublished paper, Federal Reserve Bank of San Franscisco.

Geithner, T. (2007): Remarks by New York Federal Reserve President Timothy Geithner at the Council on Foreign Relations' C. Peter McColough Roundtable Series on International Economics, New York City, January 11, 2007.

Glaeser, E. L., J. Gyourko, and R. Saks (2005): "Why Have Housing Prices Gone Up?," *American Economic Review Papers and Proceedings*, 95(2), 329–333.

Gomes, F., and A. Michaelides (2008): "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents," *Review of Financial Studies*, 21(1), 415–448.

Gopinath, G., and B. Neiman (2011): "Trade Adjustment and Productivity in Large Crises," Unpublished paper, Harvard University.

Gourinchas, P.-O., and H. Rey (2007): "From World Banker to Venture Capitalists: US External Adjustment and Exhorbitant Privilege," in *G-7 Current Account Imbalances: Sustainability and Adjustment*, ed. by R. Clarida, pp. 11–55. North Holland, Chicago, USA.

Hansen, G. D. (1985): "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309–327.

Hodrick, R., and E. C. Prescott (1997): "Post-War U.S. Business Cycles: A Descriptive Empirical Investigation," *Journal of Money, Credit, and Banking*, 29, 1–16.

Kohn, D. L. (2002): "Panel: Implications of Declining Treasury Debt. What Should the Federal Reserve Do as Treasury Debt Is Repaid?," *Journal of Money, Credit and Banking*, 34(3), 941–945.

Krishnamurthy, A., and A. Vissing-Jorgensen (2007): "The Demand for Treasury Debt," NBER Working Paper 12881.

Krusell, P., and A. A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Kydland, F., and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 343–360.

Maggiori, M. (2011): "Financial Intermediation, International Risk Sharing, and Reserve Currencies," Unpublished paper, UC Berkeley.

Mendoza, E. (2010): "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, 100(5), 1941–66.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull (2009): "Financial Integration, Financial Deepness and Global Imbalances," *Journal of Political Economy*, 117(3), 371–416.

Obstfeld, M., and K. Rogoff (2009): "Global Imbalances and the Financial Crisis: Products of Common Causes," Unpublished Paper, Harvard University.

Ríos-Rull, J., and V. Sánchez-Marcos (2006): "An Aggregate Economy with Different Size Houses," Unpublished paper, University of Minnesota.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): "Cyclical Dynamics in Idiosyncratic Labor-Market Risk," *Journal of Political Economy*, 112(3), 695–717.

——— (2007): "Asset pricing with idiosyncratic risk and overlapping generations," *Review of Economic Dynamics*, 10(4), 519–548.

Tuzel, S. (2009): "Corporate Real Estate Holdings and the Cross Section of Stock Returns," Unpublished paper, Marshall School, University of Southern California.

Vissing-Jorgensen, A. (2002): "Towards and Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures," Unpublished manuscript, Northwestern University, Kellog School of Management.

Warnock, F. E., and V. C. Warnock (2009): "International Capital Flows and U.S. Interest Rates," *Journal of International Money and Finance*, 28, 903–919.

Figure 1: Foreign Holdings of U.S. Treasuries and Agencies

Panel A plots foreign holdings of U.S. Treasuries and U.S. Agencies. U.S. Agencies denotes both the corporate bonds issued by the Government Sponsored Enterprizes and the mortgage-backed securities guaranteed by them. The solid lines denote the amount of long-term and short-term holdings, in billions of U.S. dollars, as measured against the left axis. The dashed lines denote the long-term foreign holdings relative to the total amount of outstanding long-term debt for Treasuries (squares) and Agencies (circles). The data are from the Bureau of Economic Analysis, U.S. Net International Investment Position (Table 1) and the Treasury International Capital System's annual survey of foreign portfolio holdings of U.S. securities. The data are quarterly from 1984.Q4 until 2013.Q2. Panel B plots foreign holdings of U.S. Treasuries and Agencies relative to U.S. trend GDP (solid line, squares). Trend GDP is computed with a Hodrick-Prescott filter. The dashed line (stars) asks what the foreign holdings relative to trend GDP would have been if the foreign holdings relative to the amount of debt outstanding declined the amount they did, but the amount of debt outstanding relative to trend GDP was held at 2008 values for the years 2009 and 2010.

Panel A Panel B

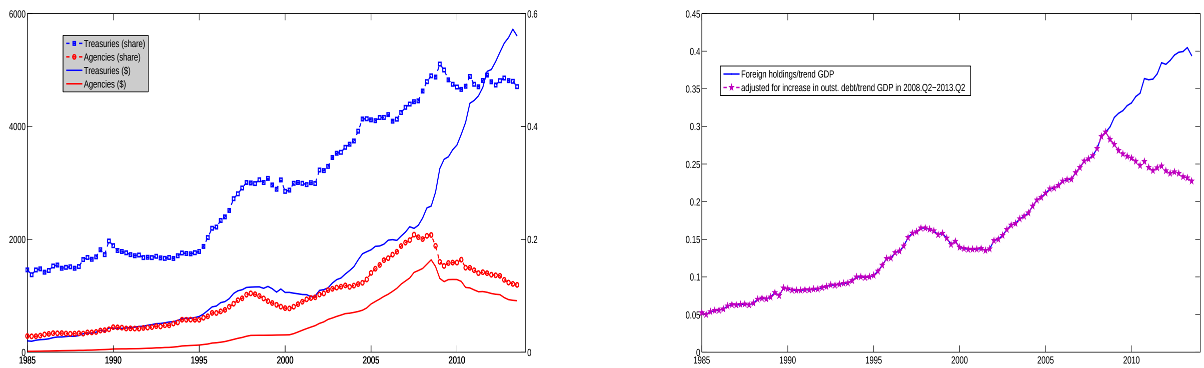

Figure 2: Net Foreign Liabilities of the U.S. Relative to U.S. Trend GDP

The solid line (squares) denotes total net foreign holdings of long-term securities (the net foreign liability position of the U.S. in those securities) relative to U.S. trend GDP. Net foreign holdings are foreign holdings of U.S. securities minus U.S. holdings of foreign securities. We define as safe the foreign holdings of U.S. Treasuries and Agencies. The dashed line (cicrles) denotes the thus constructed net foreign holdings in safe securities, while the dotted line (diamonds) denotes the net foreign holdings in all other securities. The data are from the U.S. Treasury International Capital System's annual survey of foreign portfolio holdings of U.S. securities. The data are available for December 1994, December 1997, March 2000, and annually from June 2002 until June 2010.

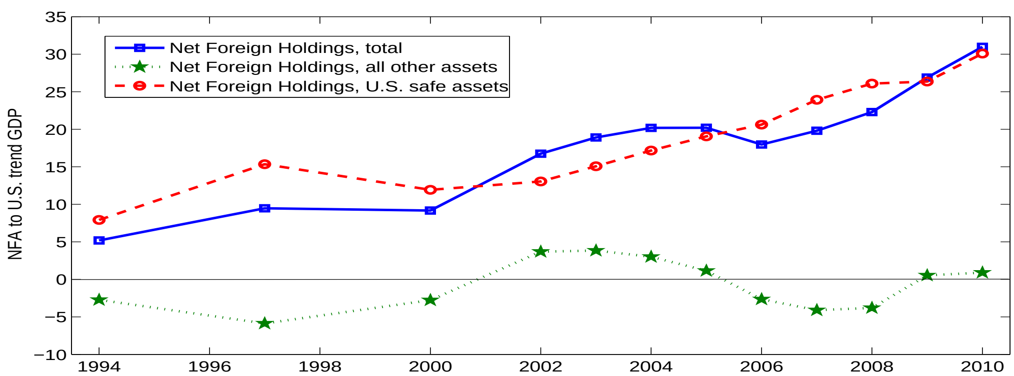

Figure 3: Welfare by Age

The left (right) panel plots the EV of an increase (decrease) in foreign holdings by age. The dashed line (circles) is the EV integrated out against the distribution of last period's bond holdings $b_{F,t}$. In particular, the dashed lines report for all age buckets (denoted by subscript a) the following welfare measure:

$$EV_a = \int\int\int\int \left\{ \left(\frac{\bar{v}_a(b_{F,t}, b_{F,t}+\Delta, Z_t^i, w_t^i, h_t^i)}{\bar{v}_a(b_{F,t}, b_{F,t}, Z_t^i, w_t^i, h_t^i)} \right)^{\frac{\sigma}{\sigma-1}} f\left(b_{F,t}\right) dZ_t^i dw_t^i dh_t^i db_{F,t} \right\},$$

where the change Δ in foreign holdings relative to trend GDP is set to equal $\Delta = (1-\rho_F)\,\overline{b} + \rho_F b_{F,t} + \sigma_F \cdot 1$ (increase) or $\Delta = (1-\rho_F)\,\overline{b} + \rho_F b_{F,t} + \sigma_F \cdot (-1)$ (decrease). Under the calibration discussed in the text, $\Delta = 2.16\%$ in quintile 1, 1.81% at the average, and 1.42% in quintile 5 in the left panel, and -1.51% in quintile 1, -1.89% at the average, and -2.25% in quintile 5 in the right panel. The solid line with squares (dotted line with diamonds) in each panel is the EV when the previous-period's holdings $b_{F,t}$ are in the lowest quintile b_{F1} (highest quintile b_{F5}). I.e., the solid line in the left panel reports

$$EV_{a|b_{F1}} = \int\int\int\int \left\{ \left(\frac{\bar{v}_a(b_{F,t}, b_{F,t}+\Delta, Z_t^i, w_t^i, h_t^i)}{\bar{v}_a(b_{F,t}, b_{F,t}, Z_t^i, w_t^i, h_t^i)} \right)^{\frac{\sigma}{\sigma-1}} f\left(b_{F,t}|b_{F,t} \in b_{F1}\right) dZ_t^i dw_t^i dh_t^i db_{F,t} \right\}.$$

and analogously for a decline in foreign holdings in the right panel ($-\Delta$ instead of $+\Delta$). The age buckets are 21-30, 31-40, 41-50, 51-60, 61-70, 71-80, 81 and above.

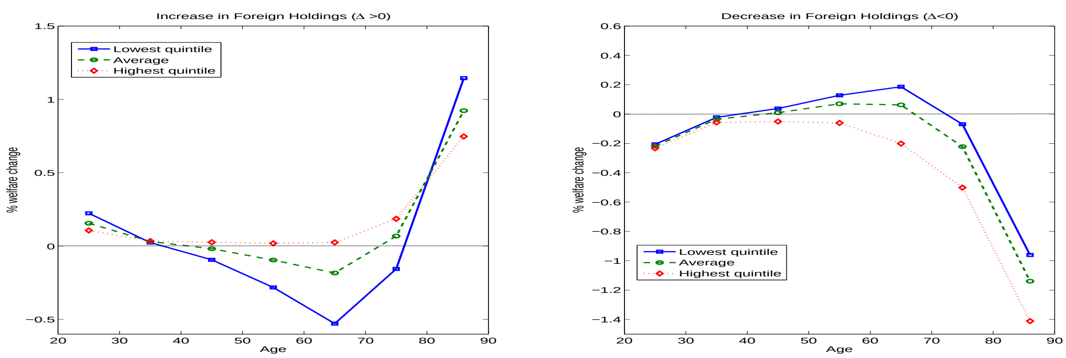

Figure 4: Welfare by Age, Income, and Financial Wealth

The left (right) panel plots the EV of a *decline* in foreign holdings by age for various income (net worth) groups. The solid line (squares) in the left (right) panel is the EV for those households in a given age bucket that are in the lowest one-third of income (net worth). The dashed line (circles) is the EV for those households in a given age bucket that are in the middle one-third of income (wealth). The dotted line (diamonds) is the EV for those households in a given age bucket that are in the highest one-third of income (wealth). The EV integrates out against the distribution of current-period and previous-period foreign bond holdings. The age buckets are 21-30, 31-40, 41-50, 51-60, 61-70, 71-80, 81 and above.

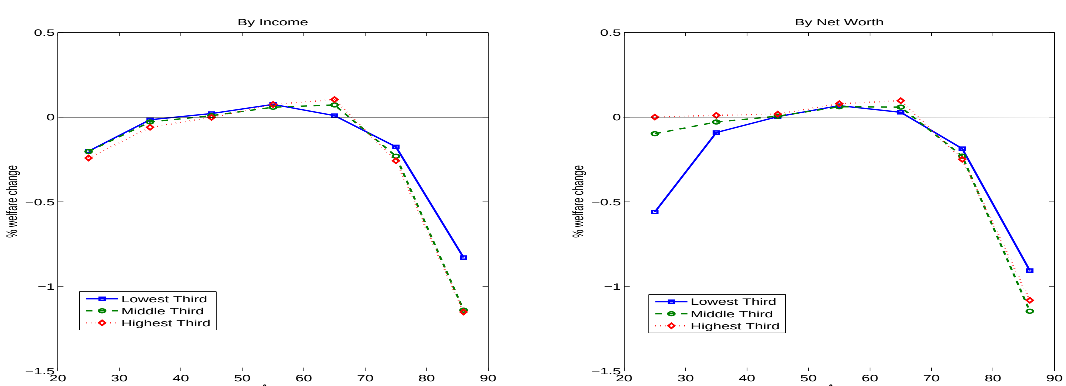

Figure 5: Welfare for Newborns

The figure plots the EV of a large *decline* in foreign holdings for a newborn (under the veil of ignorance). The measure compares the value function of a newborn, born in the 5th quintile of current foreign holdings $b_{F,t+1} = b_5$, to the welfare of a newborn, born in a world with current holdings in each of the other quintiles $b_{F,t+1} = b_{Fi}$, $for\, i = 1, \cdots, 5$ (indicated by diamonds): $EV_0(b_i) = \int \left\{ \left(\frac{\bar{v}_0(b_{F,t}, b_{Fi}, 1, 0, \bar{h}_0)}{\bar{v}_0(b_{F,t}, b_{F5}, 1, 0, \bar{h}_0)} \right)^{\frac{\sigma}{\sigma-1}} - 1 \right\} f(b_{F,t})\, db_{F,t}$, where $\bar{h}_0$ is the age-0 housing wealth agents are born with (the lowest point on the housing grid). The EV integrates out against the distribution (f) of previous-period holdings $b_{F,t}$. The fifth point (most to the right) is 0 by construction.

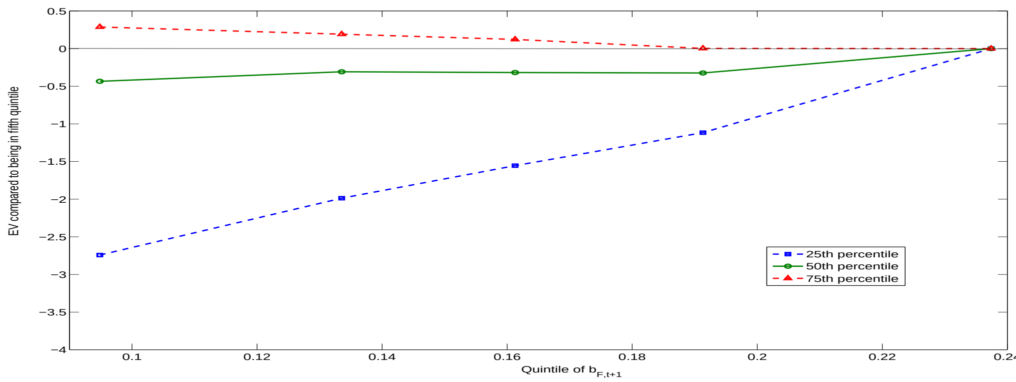

Table 1: Granger Causality

The table reports results from Granger Causality regressions of changes in the U.S. net liability position, relative to trend GDP ("flows"), on either GDP growth, or total factor productivity growth, or growth in the real exchange rate. The inverted Granger Causality regressions for GDP/TFP/exchange rate growth on flows are also reported. The regression uses overlapping quarterly observations for a sample that runs from 1984.IV until 2013.II (107 quarterly observations after adjusting the endpoints). The first column reports the point estimates from a regressions of capital inflows into safe U.S. assets ("flows"), measured as $\log(b_{F,t}) - \log(b_{F,t-4})$, on a constant, its own lag $\log(b_{F,t-4}) - \log(b_{F,t-8})$, and lagged log real gross domestic product (GDP) growth $\log(Y_{t-4}) - \log(Y_{F,t-8})$. The number in brackets are Newey-West (HAC) adjusted t-statistics with 4 lags. The last row reports the adjusted R^2, in percentage points. The second column replaces 4-quarter log changes in GDP by 4-quarter log changes in total factor productivity (TFP), while the third column replaces it by 4-quarter log changes in variable capacity-adjusted total factor productivity (TFPU). Both TFP series are from Fernald and Natsuki (2012). Columns 5 to 8 report results for regressions with the same right-hand side variables but GDP growth, TFP changes, and TFPU changes on the left-hand side instead of capital inflows.

	flows	flows	flows	flows	GDP	TFP	TFPU
constant	0.060	0.045	0.040	0.093	0.015	0.008	0.009
	[3.33]	[2.63]	[2.39]	[6.22]	[2.66]	[2.11]	[2.57]
lagged flows	0.383	0.386	0.402		0.014	0.012	0.006
	[2.68]	[2.57]	[2.49]		[0.39]	[0.42]	[0.22]
lagged GDP	-0.643			-0.818	0.351		
	[-1.15]			[-1.25]	[2.30]		
lagged TFP		-0.355				0.063	
		[-0.66]				[0.33]	
lagged TFPU			0.099				-0.033
			[0.17]				[-0.22]
adj. R^2 (%)	17.6	15.3	14.8	3.7	10.6	-1.2	-1.7

Table 2: Real Business Cycle Moments

Panel A denotes business cycle statistics in annual post-war U.S. data (1953-2012). The data combine information from NIPA Tables 1.1.5, 2.1, and 2.3.5. Output ($Y = Y_C + p^H Y_H + C_H$) is gross domestic product minus net exports minus government expenditures. Total consumption (C_T) is total private sector consumption (housing and non-housing). Housing consumption ($C_H = \mathcal{R} * H$) is consumption of housing services. Non-housing consumption (C) is total private sector consumption minus housing services. Housing investment ($p^H Y_H$) is residential investment. Non-housing investment (I) is the sum of private sector non-residential structures, equipment and software, and changes in inventory. Total investment is denoted I_T (residential and non-housing). For each series in the data, we first deflate by the disposable personal income deflator. We then construct the trend with a Hodrick-Prescott (1980) filter with parameter $\lambda = 100$. Finally, we construct detrended data as the log difference between the raw data and the HP trend, multiplied by 100. The standard deviation (first column) and correlation with GDP (second column) are based on these detrended series. The share of GDP (third column) is based on the raw data. Panel B denotes the same statistics for the benchmark model.

	Panel A: Data (1953-2012)		
	st.dev.	corr. w. GDP	share of gdp
Y	3.00	1.00	1.00
C_T	1.90	0.92	0.80
C	2.14	0.92	0.66
C_H	1.45	0.55	0.14
I_T	8.84	0.93	0.20
I	9.07	0.82	0.14
$p^H Y_H$	13.95	0.77	0.06
	Panel B: Model		
	st.dev.	corr. w. GDP	share of gdp
Y	2.66	1.00	1.00
C_T	1.69	0.93	0.70
C	1.59	0.93	0.49
C_H	1.96	0.91	0.21
I_T	9.46	0.77	0.30
I	9.60	0.77	0.24
$p^H Y_H$	13.27	0.51	0.06

Table 3: Quantities by Foreign Holdings

The table reports the first and second moments of real quantities by level of and changes in foreign holdings in the model. The quantity variables are as defined in Table 2. Panel A reports means of the quantity variables (raw, detrended data), whereas Panel B reports standard deviations (HP filtered data). In each panel, the "all" column reports the unconditional moment from a long simulation. The column "high $B_{F,t}$" ("low $B_{F,t}$") reports the conditional moment of the dated-t variable, conditional on the foreign holdings level $b_{F,t}$, which was chosen at time $t-1$, being in the highest (lowest) 1/2 of observations on the level of foreign holdings in the same long simulation. The column "high ΔB_F" ("low ΔB_F") reports the conditional moment of the dated-t variable, conditional on the foreign holdings change $\Delta b_F = b_{F,t+1} - b_{F,t}$, which is known at time t, being in the highest (lowest) 1/2 of observations on the change in foreign holdings in the same long simulation.

	Panel A: Mean					Panel B: Standard deviation				
	all	$b_{F,t}$ H	$b_{F,t}$ L	Δb_F H	Δb_F L	**all**	$b_{F,t}$ H	$b_{F,t}$ L	Δb_F H	Δb_F L
Y	2.20	2.21	2.19	2.21	2.19	2.66	2.69	2.63	2.64	2.66
C_T	1.54	1.54	1.54	1.55	1.53	1.69	1.77	1.60	1.52	1.61
C	1.07	1.07	1.07	1.07	1.06	1.59	1.68	1.50	1.42	1.52
C_H	0.47	0.47	0.47	0.47	0.47	1.96	2.02	1.89	1.81	1.86
I_T	0.66	0.67	0.65	0.70	0.61	9.46	9.92	8.94	6.53	7.51
I	0.52	0.53	0.52	0.56	0.49	9.60	10.17	8.97	7.09	7.86
$p^H Y_H$	0.13	0.14	0.13	0.14	0.12	13.27	13.18	13.32	10.43	11.36

Table 4: Sensitivity to Changes in Foreign Holdings

The second and third rows of each panel report the slope coefficients $\beta_{F,t}$ and $\beta_{\Delta b_F}$ of a multiple regression of the log change in a variable between t+1 and t on a constant, the foreign holdings level $b_{F,t}$ (chosen in period t-1), and the foreign holdings flow between t and t+1 $\Delta b_{F,t+1}$:

$$\log X_{t+1} - \log X_t = \alpha + \beta_F b_{F,t} + \beta_{\Delta b_F} \Delta b_{F,t+1} + \epsilon_t.$$

The constant in the regression is omitted. The columns in Panel A refer to the same real variables defined in Table 2 (not HP detrended). The left-hand side variables corresponding to the results reported in Panel B are the growth rate in the beginning-of-period capital ($\log K_{t+2} - \log K_{t+1}$) and housing stock ($\log H_{t+2} - \log H_{t+1}$), respectively, the aggregate wage, the aggregate house value, and the aggregate stock market (mutual fund) capitalization.

Var	**Panel A: RBC Moments**						
	Y	C_T	C	C_H	I_T	I	$p^H Y_H$
β_F	-0.00	-0.02	-0.02	-0.03	-0.20	-0.18	-0.24
$\beta_{\Delta b_F}$	0.21	0.41	0.41	0.41	4.23	4.06	4.88
Var	**Panel B: Other quantities and prices**						
	K	H	$\mathcal{W}$	$p^H H$	V		
β_F	0.00	0.01	0.00	-0.08	-0.07		
$\beta_{\Delta b_F}$	0.48	0.08	0.15	2.01	1.92		

Table 5: Asset Pricing Moments

The second column (data 1) reports the observed asset pricing moments, listed in the first column, in annual 1953-2012 data. The equity return R_S is the value-weighted CRSP stock market return minus the realized inflation rate over the course of the year. The risk-free rate is measured as the nominal yield on a one-year government bond from the CRSP Fama-Bliss data set in the last month of the preceding year minus the realized inflation rate over the course of the year. The price deflator is the same as in Table 2. The housing return R_H in data1 is the aggregate value of residential real estate wealth in the fourth quarter of the year from the Flow of Funds plus the consumption of housing services summed over the four quarters of the year from NIPA divided by the value of residential real estate in the fourth quarter of the preceding year. We subtract inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities due to population growth. The third column reports moments for the annual 1976-2012 sample. The housing return in data 2 uses the seasonally adjusted repeat-sale national house price index from Core Logic and the seasonally-adjusted rental price index for shelter from the Bureau of Labor Statistics. It assumes a price-rent ratio in 1975 equal to the one in data 1. We then use the quarterly price and rent indices to construct quarterly returns and price-rent ratios over the 1976-2012 period. We construct annual returns by compounding the quarterly returns during the year. We subtract realized inflation from realized housing returns to form real housing returns. The stock return and risk-free rate in data 2 are the same as in data 1, but measured over the shorter sample. The fourth column reports the unconditional asset pricing moments from a long simulation of the model. The fifth (sixth) column reports the same time-t moments, but conditional on being in the highest 1/2 (lowest 1/2) of foreign holdings levels $b_{F,t}$, chosen at t-1. The seventh (eight) column reports the same time-t moments, but conditional on being in the highest 1/2 (lowest 1/2) of foreign holdings changes $\Delta b_{F,t}$ between t-1 and t. For example, the last column, seventh row reports the equity risk premium (the time-t expectation of the excess return between t and t+1), conditional on having experienced a foreign outflow between t-1 and t. The first and second rows reports first and second moments of the one-period risk-free rate. The third and fourth (fifth and sixth) rows report first and second moments of the unlevered (levered) physical capital return (i.e., stock market return). The seventh row reports the average excess return, i.e., in excess of the riskfree rate. The eight row reports the Sharpe ratio, defined as the average excess return divided by the standard deviation of the excess return. Rows nine through twelve report the analogous return moments for the aggregate housing market. For columns five through 8, row thirteen (fourteen) reports the change in the house price-rent ratio (stock market price-dividend ratio), measured as the percentage change relative to the "all" periods sample in column 4.

	data 1	data 2	**all**	$b_{F,t}$ H	$b_{F,t}$ L	Δb_F H	Δb_F L
1. $E[R_f]$	1.38	1.87	0.61	0.32	0.88	-3.01	4.43
2. $Std[R_f]$	2.56	2.83	4.83	4.89	4.75	2.69	3.44
3. $E[R_K]$			4.27	4.12	4.42	0.80	7.93
4. $Std[R_K]$			7.23	7.65	6.78	6.14	6.44
5. $E[R_S]$	8.26	8.72	6.71	6.65	6.77	3.34	10.27
6. $Std[R_S]$	18.73	16.83	10.23	10.88	9.54	9.50	9.76
7. $E[R_S - R_f]$	6.88	6.85	6.10	6.33	5.89	6.35	5.83
8. $SR[R_S]$	0.38	0.42	0.67	0.66	0.69	0.71	0.63
9. $E[R_H]$	10.83	9.23	8.67	8.53	8.81	5.22	12.32
10. $Std[R_H]$	5.60	6.99	7.77	7.89	7.64	6.63	7.20
11. $E[R_H - R_f]$	9.45	7.54	8.06	8.21	7.93	8.23	7.89
12. $SR[R_H]$	1.57	0.98	1.33	1.33	1.34	1.39	1.27
13. $\Delta\left(p^H/\mathcal{R}\right)$			$--$	1.24	-1.16	2.59	-2.73
14. $\Delta\left(V/D\right)$			$--$	9.23	-8.86	50.58	-53.28

Appendix: For Online Publication

This file is the online appendix to "Foreign Ownership of U.S. Safe Assets: Good or Bad?" This appendix defines the model's stochastic discount factor, spells out all equilibrium conditions, defines equity and housing returns, describes how we calibrate the stochastic shock processes in the model, as well as how we calibrate all other parameters. The last section details our numerical solution strategy.

Stochastic Discount Factor

The stochastic discount factor (SDF), $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$, appears in the dynamic value maximization problem (2) and (3) undertaken by each representative firm. We assume that the representative firm discounts future profits using a weighted average of the individual shareholders' MRS in non-housing consumption, $\frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}}$, where the weights, $\theta^i_{a,t}$, correspond to the shareholder's proportional ownership in the firm. Let $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$ denote this weighted average:

$$\frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} = \beta \left(\frac{C^i_{a+1,t+1}}{C^i_{a,t}}\right)^{-\frac{1}{\sigma}} \left[\frac{\left(\frac{H^i_{a+1,t+1}}{C^i_{a+1,t+1}}\right)^{(1-\chi)\left(1-\frac{1}{\sigma}\right)}}{\left(\frac{H^i_{a,t}}{C^i_{a,t}}\right)^{(1-\chi)\left(1-\frac{1}{\sigma}\right)}}\right]$$

$$\frac{\beta \Lambda_{t+1}}{\Lambda_t} \equiv \int_{\mathcal{S}} \theta^i_{a+1,t+1} \frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} d\mu, \tag{A1}$$

Since we weight each individual's MRS by its proportional ownership (and since short-sales in the risky asset are prohibited), only those households who are long in the risky asset (shareholders) will receive non-zero weight in the SDF. We check that our equilibrium is not quantitatively sensitive to this assumption on ownership control. This insensitivity is predicted in a wide class of incomplete markets models–see FVLN for more discussion.

Equilibrium Conditions

The following are a complete set of equilibrium conditions:

1. Households optimize:

$$\begin{aligned}\upsilon_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t}) &= \max_{C^i_{a+1,t+1}, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}} \{U(C^i_{a,t}, H^i_{a,t}) \\ &\quad + \beta\pi_{a+1|a} E_t[\upsilon_{a+1}(\mu_{t+1}, Z_{t+1}, Z^i_{a,t+1}, W^i_{a+1,t+1}, H^i_{a+1,t+1})]\}\end{aligned} \tag{A2}$$

subject to (5), (6), if the individual of working age, and subject to (6) and the analogous versions of (5) (using pension income in place of wage income), if the individual is retired.

2. Firm's maximize value: $V_{C,t}$ solves (2), $V_{H,t}$ solves (3).

3. The land/permits price p^L_t satisfies $p^L_t = (1-\phi)\, p^H_t Z^{1-\nu\phi}_{H,t} \mathcal{L}^{-\phi}_t \left(K^{\nu}_{H,t} N^{1-\nu}_{H,t}\right)^{\phi}$.

4. The land/permits market clears: $\bar{\mathcal{L}} = \mathcal{L}_t$.

5. Wages $\mathcal{W}_t = \mathcal{W}(\mu_t, Z_t)$ satisfy

$$\mathcal{W}_t = (1-\alpha)\, Z^{1-\alpha}_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} \tag{A3}$$

$$\mathcal{W}_t = (1-\nu)\, \phi p^H_t Z^{1-\nu\phi}_{H,t} \mathcal{L}^{1-\phi}_t K^{\nu\phi}_{H,t} N^{\phi(1-\nu)-1}_{H,t}. \tag{A4}$$

6. The housing market clears: $p^H_t = p^H(\mu_t, Z_t)$ is such that

$$Y_{H,t} = \int_{\mathcal{S}} \left(H^i_{a,t+1} - H^i_{a,t}(1-\delta_H)\right) d\mu. \tag{A5}$$

7. The bond market clears: $q_t = q(\mu_t, Z_t)$ is such that

$$\int_{\mathcal{S}} B^i_{a,t} d\mu + B_{F,t} + B_{G,t} = 0. \tag{A6}$$

8. The risky asset market clears: $R_{K,t} = R_K(\mu_t, Z_t)$ is such that

$$1 = \int_{\mathcal{S}} \theta^i_{a,t} d\mu. \tag{A7}$$

9. The labor market clears:

$$N_t \equiv N_{C,t} + N_{H,t} = \int_{\mathcal{S}} L^i_{a,t} d\mu. \tag{A8}$$

10. The social security tax rate is set so that total taxes equal total retirement benefits:

$$\tau N_t \mathcal{W}_t = \int_{\mathcal{S}} PE^i_{a,t} d\mu. \tag{A9}$$

11. Government revenues from land/permits plus new debt issuance plus lump sum taxes equal government debt to be paid back this period:

$$p^L_t \mathcal{L}_t - B^G_{t+1} q_t - T_t = -B^G_t.$$

12. The presumed law of motion for the state space $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ is consistent with individual behavior.

Equations (A3), (A4) and (A8) determine the $N_{C,t}$ and therefore determine the allocation of labor across sectors:

$$(1-\alpha) Z_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} = (1-\nu)\phi p^H_t Z^{1-\nu\phi}_{H,t} \mathcal{L}^{1-\phi}_t K^{\nu\phi}_{H,t} (N_t - N_{C,t})^{\phi(1-\nu)-1}. \tag{A10}$$

Housing and Equity Returns

The first-order condition for optimal housing choice with non-binding borrowing constraints and no transactions costs is

$$U_{C^i_{a,t}} = \frac{1}{p^H_t} \beta E_t \left[U_{C^i_{a+1,t+1}} \left(\frac{U_{H^i_{a+1,t+1}}}{U_{C^i_{a+1,t+1}}} + p^H_{t+1}(1-\delta_H) \right) \right], \tag{A11}$$

where the partial derivative $\frac{\partial U}{\partial C^i_{a,t}}$ is written $U_{C^i_{a,t}}$, and analogously for $U_{C^i_{a+1,t+1}}$ and $U_{H^i_{a+1,t+1}}$. Each individual's housing return is given by $\left(U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}} + p^H_{t+1}(1-\delta_H)\right)/p^H_t$ where $U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}}$ is a measure of fundamental value, the service flow value generated by the housing asset. Binding borrowing constraints change (A11) from an equality to an inequality and transactions costs add additional terms to the price term $p^H_{t+1}(1-\delta_H)$, but neither of these change the definition of the housing service flow or the definition of the individual return. In a competitive equilibrium, $U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}}$ is equal to the relative price of housing services. For brevity, we refer to this quantity hereafter as "rent," but it should be kept in mind that it is actually a measure of the flow dividend from the housing asset for owner-occupied housing. The addition of an explicit rental market would make the numerical solution intractable, given the existing complexity.

To obtain the model's implications for a national housing return, computed from an aggregate house price index combined with an aggregate housing service flow index (also quantities readily observable in aggregate data), we form an aggregate (across households) measure of the individual housing service flows and refer to it as "national rent," denoted $\mathcal{R}_{t+1}$. In the model, p_t^H is the price of a unit of housing stock, which holds fixed the composition of housing (quality, square footage, etc.) over time. It is the same for everyone, thus it the model-based national house price index, akin to a repeat-sale index in the data. We combine $\mathcal{R}_{t+1}$ with the national house-price index p_{t+1}^H to compute a corresponding *national housing index return* $R_{H,t+1}$:

$$R_{H,t+1} \equiv \frac{p_{t+1}^H (1-\delta_H) + \mathcal{R}_{t+1}}{p_t^H}, \tag{A12}$$

$$\mathcal{R}_{t+1} \equiv \int_{\mathcal{S}} \frac{U_{H^i_{a+1,t+1}}}{U_{C^i_{a+1,t+1}}} d\mu. \tag{A13}$$

We refer to $p_{t+1}^H / \mathcal{R}_{t+1}$, as the national "price-rent" ratio for brevity. We also compute the standard deviation of the return on the housing index return (A12), $Std\left[R_{H,t+1}\right]$, and the ratio of the time-series mean excess return on this index, divided by the standard deviation,

$$SR\left[R_H\right] \equiv \frac{E\left[R_{H,t+1} - R_{f,t+1}\right]}{Std\left[R_{H,t+1} - R_{f,t+1}\right]}.$$

This latter quantity is denoted "$SR\left[R_H\right]$" to recall the familiar "Sharpe Ratio" concept in finance, but it should be emphasized that the corresponding measure here does not represent an actual risk-return tradeoff that a household could earn, because it ignores the effects of housing transactions costs and binding borrowing constraints. In addition, in this case the statistic pertains to the return on an aggregate house price index, which is not a tradeable asset. Thus, what is denoted "SR" here is just another aggregate statistic that can be compared across model and data, not representative of a true risk-return tradeoff. These measures are, however, comparable to analogous objects constructed from aggregate data using national house price indexes and national rent or housing service flow indexes.

Risky Asset and Equity Return

The firms' values $V_{H,t}$ and $V_{C,t}$ are the cum-dividend values, measured before the dividend is paid out. Thus the cum-dividend returns to shareholders in the housing sector and the consumption sector are defined, respectively, as

$$R_{Y_H,t+1} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \qquad R_{Y_C,t+1} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.$$

We define $V_{j,t}^e = V_{j,t} - D_{j,t}$ for $j = H, C$ to be the ex-dividend value of the firm.

We define a "mutual fund" of risky assets as the value-weighted portfolio with cum-dividend return

$$R_{K,t+1} = \frac{V_{H,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_H,t+1} + \frac{V_{C,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_C,t+1}. \tag{A14}$$

The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where q_{t-1} is the bond price known at time $t-1$.

The risky capital return $R_{K,t}$ in (A14) is the return on a value-weighted portfolio of risky capital. This is not the same as the empirical return on equity, which is a levered claim on risky capital. To obtain an equity return in the model, $R_{S,t}$, the return on assets, $R_{K,t}$, must be adjusted for leverage:

$$R_{S,t} \equiv R_{f,t} + (1 + B/S)(R_{K,t} - R_{f,t}),$$

where B/S is the fixed debt-equity ratio and where $R_{K,t}$ is the portfolio return for risky capital given in (A14).[21] Note that this calculation explicitly assumes that corporate debt in the model is exogenous, and held in fixed proportion to the value of the firm. (There is no financing decision.) For the results reported below, we set $B/S = 2/3$ to match aggregate debt-equity ratios computed in Benninga and Protopapadakis (1990). This treatment of corporate leverage is standard in the finance literature.

[21] The cost of capital R_K is a portfolio weighted average of the return on debt R_f and the return on equity R_e: $R_K = aR_f + (1-a) R_e$, where $a \equiv \frac{B}{B+E}$.

Calibration of Stochastic Processes

Process for $b_{F,t}$

Individuals in the model form beliefs about the evolution of the stochastic process for foreign holdings relative to trend GDP, $b_{F,t} = B_{F,t}/\overline{Y}_t$. We assume these beliefs take the form given in (1) and calibrate parameters of this process from U.S. data. In the data, $\overline{Y}_t$ is trend GDP as computed from a Hodrick-Prescott filter (Hodrick and Prescott (1997)). The historical data on $B_{F,t}$ consist of 115 quarterly observations between the fourth quarter of 1984 and the second quarter of 2013, on foreign ownership of U.S. Treasury debt (T-bonds and T-notes). As explained in the main text and shown in Figure 1 Panel B, we adjust the series for $b_{F,t}$ to take into account that the supply of U.S. Treasuries started to rise after 2008, whereas it is constant in the model. The source for these data is Department of the Treasury, Treasury International Capital System division. The numerical grid is set to match the span of observations on $b_{F,t}$. We estimate a regression of $b_{F,t}$ on $b_{F,t-4}$. This leads to the parameter combination, $\rho_F = 0.968$, $\bar{b} = 0.1475$, and $\sigma_F = 0.017$. We use a value for the persistence parameter, $\rho_F = 0.95$, that is slightly lower than the point estimate since it delivers more stable numerical results. The innovation η_{t+1} is assumed to take on two values with equal probability: $\eta = [1, -1]$.

Calibration of Aggregate Productivity Shocks

The aggregate technology shock processes Z_C and Z_H are calibrated following a two-state Markov chain, with two possible values for each shock, $\{Z_C = Z_{Cl}, Z_C = Z_{Ch}\}$, $\{Z_H = Z_{Hl}, Z_H = Z_{Hh}\}$, implying four possible combinations:

$$\begin{aligned}
Z_C &= Z_{Cl}, \qquad Z_H = Z_{Hl} \\
Z_C &= Z_{Ch}, \qquad Z_H = Z_{Hl} \\
Z_C &= Z_{Cl}, \qquad Z_H = Z_{Hh} \\
Z_C &= Z_{Ch}, \qquad Z_H = Z_{Hh}.
\end{aligned}$$

Each shock is modeled as,

$$\begin{aligned} Z_{Cl} &= 1-e_C, \qquad Z_{Ch}=1+e_C \\ Z_{Hl} &= 1-e_H, \qquad Z_{Ch}=1+e_H, \end{aligned}$$

where e_C and e_H are calibrated to match the volatilities of GDP and residential investment in the data.

We assume that Z_C and Z_H are independent of one another. Let $\mathbf{P}^C$ be the transition matrix for Z_C and $\mathbf{P}^H$ be the transition matrix for Z_H. The full transition matrix equals

$$\mathbf{P} = \begin{bmatrix} p_{ll}^H \mathbf{P}^C & p_{lh}^H \mathbf{P}^C \\ p_{hl}^H \mathbf{P}^C & p_{hh}^H \mathbf{P}^C \end{bmatrix},$$

where

$$\mathbf{P}^H = \begin{bmatrix} p_{ll}^H & p_{lh}^H \\ p_{hl}^H & p_{hh}^H \end{bmatrix} = \begin{bmatrix} p_{ll}^H & 1-p_{ll}^H \\ 1-p_{hh}^H & p_{hh}^H \end{bmatrix},$$

and where we assume $\mathbf{P}^C$, defined analogously, equals $\mathbf{P}^H$. We calibrate values for the matrices as

$$\begin{aligned} \mathbf{P}^C &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} \\ \mathbf{P}^H &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} => \\ \mathbf{P} &= \begin{bmatrix} .36 & .24 & .24 & .16 \\ .15 & .45 & .10 & .30 \\ .15 & .10 & .45 & .30 \\ .0625 & .1875 & .1875 & .5625 \end{bmatrix}. \end{aligned}$$

With these parameter values, we match the average length of expansions divided by the average length of recessions (equal to 5.7 in NBER data from over the period 1945-2001). We define a recession as the event $\{Z_{Cl,}, Z_{Hl,}\}$, so that the probability of staying in a recession is $p_{ll}^H p_{ll}^C = 0.36$, implying that a recession persists on average for $1/(1-.36) = 1.56$ years. We define an expansion as either the event $\{Z_{Ch,} Z_{Hl,}\}$ or $\{Z_{Cl}, Z_{Hh}\}$ or $\{Z_{Ch,} Z_{Hh,}\}$. Thus, there are four possible states (one recession, three expansion). The average amount of time spent in each state is given by the stationary distribution (4×1) vector $\boldsymbol{\pi}$, where

$$\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}.$$

That is, $\boldsymbol{\pi}$ is the eigenvector for $\mathbf{P}$ with corresponding eigenvalue equal to 1. The first element of $\boldsymbol{\pi}$, denoted π_1, multiplies the probabilities in $\mathbf{P}$ for transitioning to any of the four states tomorrow conditional on being in a recession state today. π_1 therefore gives the average amount of time spent in the recession state, while π_2, π_3, and π_4 give the average amount of time spent in the other three (expansion) states. Given the matrix $\mathbf{P}$ above, the solution for $\boldsymbol{\pi}$ is

$$\boldsymbol{\pi} = \begin{pmatrix} 0.1479 \\ 0.2367 \\ 0.2367 \\ 0.3787 \end{pmatrix}.$$

This implies the chain spends 14.79% of the time in a recession state and 85.21% of the time in expansion states, so the average length of expansions relative to that of recessions is $85.21/(14.79) = 5.76$ years.

Calibration of Idiosyncratic Productivity Shocks

Idiosyncratic income shocks follow the first order Markov process $\ln\left(Z^i_{a,t}\right) = \ln\left(Z^i_{a-1,t-1}\right) + \epsilon^i_{a,t}$. We directly calibrate the specification in levels:

$$Z^i_{a,t} = Z^i_{a,t-1}\left(1 + E^i_{a,t}\right),$$

where $E^i_{a,t}$ takes on one of two values in each aggregate state:

$$\begin{aligned} E^i_{a,t} &= \begin{cases} \sigma_E & \text{with } \Pr = 0.5 \\ -\sigma_E & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ E^i_{a,t} &= \begin{cases} \sigma_R & \text{with } \Pr = 0.5 \\ -\sigma_R & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \\ \sigma_R &> \sigma_E. \end{aligned}$$

Thus, $E\left(Z^i_{a,t}/Z^i_{a,t-1}\right) = 1$.

Calibration of Parameters

This appendix describes in detail how we choose all other parameters of the model. All parameter values are summarized in the table at the end of this subsection.

Parameters pertaining to the firms' decisions are set as follows. The capital depreciation rate, δ, is set to 0.12, which corresponds to the average Bureau of Economic Analysis (BEA) depreciation rates for equipment and structures. The housing depreciation rate δ_H, is set to 0.025 following Tuzel (2009). Following Kydland and Prescott (1982) and Hansen (1985), the capital share for the non-housing sector is set to $\alpha = 0.36$. For the residential investment sector, the value of the capital share in production is taken from a BEA study of gross product originating, by industry. The study finds that the capital share in the construction sector ranges from 29.4% and 31.0% over the period 1992-1996. We therefore set the capital share in the housing sector to $\nu = 0.30$. The adjustment costs for capital in both sectors are assumed to be the same quadratic function of the investment to capital-ratio, $\varphi\left(\frac{I}{K} - \delta\right)^2$, where the constant φ is chosen to represent a tradeoff between the desire to match aggregate investment volatility simultaneously with the volatility of asset returns. Under this calibration, firms pay a cost only for net new investment; there is no cost to replace depreciated capital. This implies that the total adjustment cost $\varphi\left(\frac{I}{K} - \delta\right)^2 K_t$ under our calibration is quite small: on average less than one percent of investment, I_t. The fixed quantity of land/permits available each period, $\bar{\mathcal{L}}$, is set to a level that permits the model to approximately match the housing investment-GDP ratio. In post-war data this ratio is 6%; under our calibration of $\bar{\mathcal{L}}$, the ratio ranges from 5% to 6.2%. The share of land/permits in the housing production function is set to 10%, to match estimates in Davis and Heathcote (2005), requiring $\phi = 0.9$.

Parameters of the individual's problem are set as follows. The subjective time discount factor is set to $\beta = 0.976$ at annual frequency, to allow the model to match the mean of a short-term Treasury rate in the data. The survival probability $\pi_{a+1|a} = 1$ for $a+1 \leq 65$. For $a+1 > 65$, we set $\pi_{a+1|a}$ equal to the fraction of households over 65 born in a particular year alive at age $a+1$, as measured by the U.S. Census Bureau. From these numbers, we obtain the stationary age distribution in the model, and use it to match the average earnings over the life-cycle, G_a, to that observed from the Survey of Consumer Finances. Risk aversion is set to $\sigma^{-1} = 8$, to help the models match the high Sharpe ratio for equity observed in the data. The weight, χ on C in the utility function is set to 0.70, corresponding to a housing expenditure share of 0.30. The regime-switching conditional variance in the unit root process in idiosyncratic earnings is calibrated following Storesletten, Telmer, and Yaron (2007) to match their estimates from the Panel Study of Income Dynamics. These are $\sigma_E = 0.0768$, and $\sigma_R = 0.1296$.

To calibrate the costs of equity market participation we follow results in Vissing-Jorgensen (2002), who finds support for the presence of a fixed, per period participation cost, but not for the hypothesis of variable costs. She estimates the size of these costs and finds that they are small, less than 50 dollars per year in year 2000 dollars. These findings motivate our calibration of these costs so that they are no greater than 1% of per capita, average consumption, denoted $\overline{F}^i$ in Table 2.

We set the maximum combined LTV (first and second mortgages) to be 75%, corresponding to $\varpi = 25\%$. It should be emphasized that $1 - \varpi$ gives the *maximum* combined (first and second mortgage) LTV ratio. This will differ from the *average* LTV ratio because not everyone borrows up to the credit limit.

The fixed and variable housing transactions costs for housing consumption are governed by the parameters ψ_0 and ψ_1. These costs are more comprehensive than the costs of buying and selling existing homes. They include costs of any change in housing consumption, such as home improvements and additions, as well as non-pecuniary psychological costs. We set the values of fixed costs ψ_0 and variable costs ψ_1 to be half-way between the values specified in Model 1 and Model 3 of Favilukis, Ludvigson, and Van Nieuwerburgh (2008). The Model 1 parameter values in that paper were intended to match "normal times," a period prior to the housing boom of 2000-2006. Model 3 parameters are calibrated to match evidence that transactions costs for obtaining housing finance had declined, thus lowering these parameters. Recent existing evidence suggests we have at least partially reverted to the Model 1 parameter values, in the aftermath of the credit crisis (see Favilukis, Kohn, Ludvigson, and Van Nieuwerburgh (2013)). Thus we set parameters in between the two calibrations. The Model 1 parameter values are anchored by setting the average number of years that individuals in the model go without changing housing consumption equal to the average length of residency (in years) for home owners in the Survey of Consumer Finances across the 1989-2001 waves of the survey. This leaves a value for ψ_0 that is approximately 3.2% of annual per capita consumption, and a value for ψ_1 that is approximately 5.5% of the value of the house $p_t^H H_{a,t}^i$.

Similarly, we set the lending cost parameter, $\lambda = 4.5\%$ to be half-way between the values specified in Model 1 and Model 3 of Favilukis, Ludvigson, and Van Nieuwerburgh (2008). Their estimates come from the Federal Housing Financing Board, which reports data on mortgage origination and refinancing costs from a survey of lenders and are also consistent with data on fees pre dollar of real estate loans.

We set $b^G < 0$ equal to the (negative of) the observed ratio of government debt held by the public to trend GDP over the period 1984-2008, which is equal to 30%.

The decomposition of the population into workers and retirees is determined from life-expectancy tables as follows. Let X denote the total number of people born each period. (In practice this is calibrated to be a large number in order to approximate a continuum.) Then $N^W = 45 \cdot X$ is the total number of workers. Next, from life expectancy tables, if the probability of dying at age $a > 45$ is denoted p_a then $N^R = \sum_{a=46}^{80} (1 - p_a) X$ is the total number of retired persons.

Calibration

	Parameter	Description	Value
		Production	
1	φ	cap. adjustment cost	4
2	δ	deprec., K_C, K_H	12% p.a.
3	δ_H	depreciation, H	2.5% p.a.
4	α	capital share, Y_C	0.36
5	ν	capital share, Y_H	0.3
6	ϕ	non-land share, Y_H	0.9
		Preferences	
7	σ^{-1}	risk aversion	8
8	χ	weight on C	0.70
9	β	time disc factor	0.976
		Demographics and Income	
10	G_a	age earnings profile	SCF
11	$\pi_{a+1\|a}$	survival prob	mortality tables
12	g	deterministic growth of economy	0.02
13	σ_E	st. dev. ind earnings, E	0.0768
14	σ_R	st. dev. ind earnings, R	0.1298
		Transactions Costs	
15	$\overline{F}$	participation cost, K	$\approx 1\%\ \overline{C}^i$
16	ψ_0	fixed trans cost, H	$\approx 3.2\%\ \overline{C}^i$
17	ψ_1	variable trans cost, H	$\approx 5.5\%\ p_t^H H^i$
18	ϖ	collateral constr	25%
19	λ	borrowing cost	5.5%
		Government Borrowing	
20	b^G	safe debt net supply/trend GDP	0.30
		Foreign Holdings of Safe Debt	
21	$\bar{b}$	mean for. holdings/trend GDP	0.148
22	ρ_F	persistence for. holdings/trend GDP	0.95
23	σ_F	innovation volatility for.holdings/trend GDP	1.7%

Numerical Solution Procedure

The numerical solution strategy consists of solving the individual's problem taking as given her beliefs about the evolution of the aggregate state variables. With this solution in hand, the economy is simulated for many individuals and the simulation is used to compute the equilibrium evolution of the aggregate state variables, given the assumed beliefs. If the equilibrium evolution differs from the beliefs individuals had about that evolution, a new set of beliefs are assumed and the process is repeated. Individuals' expectations are rational once this process converges and individual beliefs coincide with the resulting equilibrium evolution. One important note: we have no results on uniqueness. We are unaware of any such results in the literature concerning models with the degree of complexity considered here, as is typically the case.

The state of the economy is a pair, (Z_t, μ_t), where μ_t is a measure defined over

$$\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathfrak{W} \times \mathcal{H}),$$

where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathfrak{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ_t is a distribution of agents across ages, idiosyncratic shocks, financial, and housing wealth. $Z_t = (Z_{C,t}, Z_{H,t}, B_{F,t}, B_{F,t})$ is a vector containing all exogenous state variables. Given a finite dimensional vector to approximate μ_t, and a vector of individual state variables

$$\mu_t^i = (z_t^i, w_t^i, h_t^i),$$

the individual's problem is solved using dynamic programming.

An important step in the numerical strategy is approximating the joint distribution of individuals, μ_t, with a finite dimensional object. The resulting approximation, or "bounded rationality" equilibrium has been used elsewhere to solve overlapping generations models with heterogenous agents and aggregate risk, including Krusell and Smith (1998); Ríos-Rull and Sánchez-Marcos (2006); Storesletten, Telmer, and Yaron (2007); Gomes and Michaelides (2008); Favilukis (2013), among others. the aggregate resource constraint for this economy implies that the net *change* in the value of foreign capital, or the trade balance, influences current spending relative to current resources. For this reason, both $B_{F,t+1}$ and $B_{F,t}$ are aggregate state variables as of time t. For our application, we approximate this space with a vector of aggregate state variables other than $B_{F,t}$ and $B_{F,t+1}$ with (in detrended values)

$$\mu_t^{AG} = (z_{C,t}, z_{H,t}, k_t, S_t, h_t, p_t^H, q_t),$$

where

$$K_t = K_{C,t} + K_{H,t}$$

and

$$S_t = \frac{K_{C,t}}{K_{C,t} + K_{H,t}}.$$

The state variables are the observable aggregate technology shocks, the first moment of the aggregate capital stock, the share of aggregate capital used in production of the consumption good, the aggregate stock of housing, and the relative house price and bond price, respectively. The bond and the house price are natural state variables because the joint distribution of all individuals only matters for the individual's problem in so far as it affects asset prices. Note that knowledge of K_t and S_t is tantamount to knowledge of $K_{C,t}$ and $K_{H,t}$ separately, and vice versa ($K_{C,t} = K_t S_t$; $K_{H,t} = K_t(1 - S_t)$).

To solve the model, all variables are divided by the trend component $\exp(gt)$ to obtain policy functions and state variables have invariant distributions. In the simulations, we recover the levels of the variables by multiplying them by $\exp(gt)$ and returns are multiplied by $(1 + g)$.

Because of the large number of state variables and because the problem requires that prices in two asset markets (housing and bond) must be determined by clearing markets every period, the proposed problem is highly numerically intensive. To make the problem tractable, we obviate the need to solve the dynamic programming problem of firms numerically by instead solving analytically for a recursive solution to value function taking the form $V(K_t) = Q_t K_t$, where Q_t (Tobin's q) is a recursive function. We discuss this below.

In order to solve the individual's dynamic programming problem, the individual must know μ_{t+1}^{AG} and μ_{t+1}^i as a function of μ_t^{AG} and μ_t^i and aggregate shocks $Z_{t+1} = (Z_{C,t}, Z_{H,t}, B_{F,t}, B_{F,t+1})$. Here we show that this can be achieved by specifying individuals' beliefs for the laws of motion of four quantities:

A1 K_{t+1},

A2 p^H_{t+1},

A3 q_{t+1}, and

A4 $[\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})]$, where $Q_{C,t+1} \equiv V_{C,t+1}/K_{C,t+1}$ and analogously for $Q_{H,t+1}$.

Let $\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t} \equiv M_{t+1}$. The beliefs are approximated by a linear function of the aggregate state variables as follows:

$$\varkappa_{t+1} = A^{(n)}(Z_t, Z_{t+1}) \times \widetilde{\varkappa}_t, \tag{A15}$$

where $A^{(n)}(Z_t, Z_{t+1})$ is a 4×5 matrix that depends on the aggregate shocks Z_t, and $Z_{t+1}$and where

$$\begin{aligned} \varkappa_{t+1} &\equiv \left[K_{t+1}, p^H_{t+1}, q_{t+1}, [M_{t+1}(Q_{C,t+1} - Q_{H,t+1})]\right]', \\ \widetilde{\varkappa}_t &\equiv \left[K_t, p^H_t, q_t, S_t, H_t\right]'. \end{aligned}$$

We initialize the law of motion (A15) with a guess for the matrix $A^{(n)}(Z_t, Z_{t+1})$, given by $A^{(0)}(Z_t, Z_{t+1})$. The initial guess is updated in an iterative procedure (described below) to insure that individuals' beliefs are consistent with the resulting equilibrium.

Given (A15), individuals can form expectations of μ^{AG}_{t+1} and μ^i_{t+1} as a function of μ^{AG}_t and μ^i_t and aggregate shocks Z_{t+1}. To see this, we employ the following equilibrium relation (as shown below) linking the investment-capital ratios of the two production sectors:

$$\frac{I_{H,t}}{K_{H,t}} = \frac{I_{C,t}}{K_{C,t}} + \frac{1}{2\varphi} E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]. \tag{A16}$$

Moreover, note that $E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]$ can be computed from (A15) by integrating the 4th equation over the possible values of Z_{t+1} given $\widetilde{\varkappa}_t$ and Z_t.

Equation (A16) is derived by noting that the consumption firm solves a problem taking the form

$$V(K_{C,t}) = \max_{I_{C,t}, N_{C,t}} Z_{C,t} K^{\alpha}_{Ct} N^{1-\alpha}_{C,t} - w_t N_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 + E_t\left[M_{t+1} V(K_{C,t+1})\right].$$

The first-order condition for optimal labor choice implies $N_{C,t} = \left(\frac{Z_{C,t}(1-\alpha)}{w_t}\right)^{1/\alpha} K_{C,t}$. Substituting this expression into $V(K_{C,t})$, the optimization problem may be written

$$\begin{aligned} V(K_{C,t}) &= \max_{I_t} X_{C,t} K_{C,t} - I_{C,t} - \varphi \left(\frac{I_{C,t}}{K_{C,t}} - \delta \right)^2 K_{C,t} + E_t \left[M_{t+1} V(K_{C,t+1}) \right] \\ s.t. \quad K_{C,t+1} &= (1-\delta) K_{C,t} + I_{C,t} \end{aligned} \tag{A17}$$

where

$$X_{C,t} \equiv \alpha \left(\frac{Z_{C,t}}{w_t} (1-\alpha) \right)^{(1-\alpha)/\alpha} Z_{C,t}$$

is a function of aggregate variables over which the firm has no control.

The housing firms solves

$$\begin{aligned} V(K_{H,t}) = \max_{I_{H,t}, N_{H,t}} & p_t^H Z_{H,t} (\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu} \right)^{\phi} - w_t N_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t \\ & - \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 + E_t \left[M_{t+1} V(K_{H,t+1}) \right]. \end{aligned} \tag{A18}$$

The first-order conditions for optimal labor and land/permits choice for the housing firm imply that $N_{H,t} = k_N K_{H,t}$, $\mathcal{L}_t = k_L K_{H,t}$, where

$$\begin{aligned} k_N &= \left(k_1^{\phi} k_2^{1-\phi} \right)^{1/\nu\phi} \\ k_L &= \left(k_1^{\phi(1-\nu)} k_2^{1-\phi(1-\nu)} \right)^{1/\phi\nu} \\ k_1 &= p_t^H Z_{H,t} \phi (1-\nu) / w_t \\ k_2 &= p_t^H Z_{H,t} (1-\phi) / p_t^L. \end{aligned}$$

Substituting this expression into $V(K_{H,t})$, the optimization problem may be written

$$\begin{aligned} V(K_{H,t}) &= \max_{I_t} X_{Ht} K_{H,t} - I_{H,t} - \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 K_{H,t} + E_t \left[M_{t+1} V(K_{H,t+1}) \right] \\ s.t. \quad K_{H,t+1} &= (1-\delta) K_{H,t} + I_{H,t} \end{aligned} \tag{A19}$$

where

$$X_{H,t} = p_t^H Z_{H,t} \phi \nu k_N^{(1-\nu)\phi} k_L^{1-\phi}.$$

Let s index the sector as either consumption, C, or housing, H. We now guess and verify that for each firm, $V(K_{s,t+1})$, for $s = C, H$ takes the form

$$V(K_{s,t+1}) = Q_{s,t+1} K_{s,t+1}, \qquad s = C, H \tag{A20}$$

where $Q_{s,t+1}$ depends on aggregate state variables but is not a function of the firm's capital stock $K_{s,t+1}$ or investment $I_{s,t}$. Plugging (A20) into (A17) we obtain

$$V(K_{s,t}) = \max_{I_t} X_{s,t}K_{s,t} - I_t - \varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2 K_{s,t} + E_t\left[M_{t+1}Q_{s,t+1}\right]\left[(1-\delta)K_{s,t} + I_{s,t}\right]. \tag{A21}$$

The first-order conditions for the maximization (A21) imply

$$\frac{I_{s,t}}{K_{s,t}} = \delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right] - 1}{2\varphi}. \tag{A22}$$

Substituting (A22) into (A21) we verify that $V(K_{s,t})$ takes the form $Q_{s,t}K_{s,t}$:

$$\begin{aligned} V(K_{s,t}) \equiv\ & Q_{s,t}K_{s,t} = X_{s,t}K_{s,t} - \left(\delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)K_{s,t} - \varphi\left(\frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)^2 K_{s,t} \\ & + (1-\delta)\left(E_t\left[M_{t+1}Q_{s,t+1}\right]\right)K_{s,t} + E_t\left[M_{t+1}Q_{s,t+1}\right]\left(\delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)K_{s,t}. \end{aligned}$$

Rearranging terms, it can be shown that $Q_{s,t}$ is a recursion:

$$Q_{s,t} = X_{s,t} + (1-\delta) + \left[2\varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)\right] + \varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2. \tag{A23}$$

Since $Q_{s,t}$ is a function only of $X_{s,t}$ and the expected discounted value of $Q_{s,t+1}$, it does not depend on the firm's own $K_{s,t+1}$ or $I_{s,t}$. Hence we verify that $V(K_{s,t}) = Q_{s,t}K_{s,t}$. Although $Q_{s,t}$ does not depend on the firm's individual $K_{s,t+1}$ or $I_{s,t}$, in equilibrium it will be related to the firm's investment-capital ratio via:

$$Q_{s,t} = X_{s,t} + (1-\delta)\left[1 + 2\varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)\right] + \varphi\left(\frac{I_{s,t}}{K_{s,t}}\right)^2 - 2\varphi\delta\left(\frac{I_{s,t}}{K_{s,t}}\right), \tag{A24}$$

as can be verified by plugging (A22) into (A23). Note that (A22) holds for the two representative firms of each sector, i.e., $Q_{C,t}$ and $Q_{H,t}$, thus we obtain (A16) above.

With (A24), it is straightforward to show how individuals can form expectations of μ^{AG}_{t+1} and μ^{i}_{t+1} as a function of μ^{AG}_{t} and μ^{i}_{t} and aggregate shocks Z_{t+1}. Given a grid of values for K_t and S_t individuals can solve for $K_{C,t}$ and $K_{H,t}$ from $K_{C,t} = K_tS_t$ and $K_{H,t} = K_t(1-S_t)$. Combining this with beliefs about K_{t+1} from (A15), individuals can solve for $I_t \equiv I_{C,t} + I_{H,t}$ from $K_{t+1} = (1-\delta)K_t + I_t$. Given I_t and beliefs about $\left[\frac{\beta^k\Lambda_{t+k}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})\right]$ from (A15), individuals can solve for $I_{C,t}$ and $I_{H,t}$ from (A16). Given $I_{H,t}$ and the accumulation equation $K_{H,t+1} = (1-\delta) K_{H,t} + I_{H,t}$, individuals can solve for $K_{H,t+1}$. Given $I_{C,t}$ individuals can solve for $K_{C,t+1}$ using the accumulation equation $K_{C,t+1} = (1-\delta) K_{C,t} + I_{C,t}$. Using $K_{H,t+1}$ and $K_{C,t+1}$, individuals can solve for S_{t+1}. Given a grid of values for H_t, H_{t+1} can be computed from $H_{t+1} = (1-\delta_H) H_t + Y_{H,t}$, where $Y_{H,t} = Z_{H,t} (\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu}\right)^{\phi}$ is obtained from knowledge of $Z_{H,t}$, $K_{H,t}$ (observable today), from the equilibrium condition $\mathcal{L}_t = L$, and by combining (A8) and (A10) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Equation (A15) can be used directly to obtain beliefs about q_{t+1} and p_{t+1}^H.

To solve the dynamic programming problem individuals also need to know the equity values $V_{C,t}$ and $V_{H,t}$. But these come from knowledge of $Q_{s,t}$ (using (A24)) and $K_{s,t}$ via $V_{s,t} = Q_{s,t} K_{s,t}$ for $s = C, H$. Values for dividends in each sector are computed from

$$\begin{aligned} D_{C,t} &= Y_{C,t} - I_{C,t} - w_t N_{C,t} - \phi_C \left(\frac{I_{C,t}}{K_{C,t}}\right) K_{C,t}, \\ D_{H,t} &= p_t^H Y_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t - w_t N_{H,t} - \phi_H \left(\frac{I_{H,t}}{K_{H,t}}\right) K_{H,t} \end{aligned}$$

and from

$$w_t = (1-\alpha) Z_{j,t} K_{j,t}^{\alpha} N_{j,t}^{-\alpha} = (1-\nu)(1-\phi) p_t^H Z_{H,t} \mathcal{L}_t^{\phi} K_{H,t}^{\nu(1-\phi)} N_{H,t}^{-\phi(1-\nu)-\nu}$$

and by again combining (A8) and (A10) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Finally, the evolution of the aggregate technology shocks Z_{t+1} is given by the first-order Markov chain described above; hence agents can compute the possible values of Z_{t+1} as a function of Z_t.

Values for $\mu_{t+1}^i = (Z_{t+1}^i, W_{t+1}^i, H_{t+1}^i)$ are given from all of the above in combination with the first order Markov process for idiosyncratic income $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$. Note that H_{t+1}^i is a choice variable, while $W_{t+1}^i = \theta_t^i (V_{C,t+1} + V_{H,t+1} + D_{C,t+1} + D_{H,t+1}) + B_{t+1}^i$ requires knowing $V_{s,t+1} = Q_{s,t+1} K_{s,t+1}$ and $D_{s,t+1}$, $s = C, H$ conditional on Z_{t+1}.These in turn depend on $I_{s,t+1}$, $s = C, H$ and may be computed in the manner described above by rolling forward one period both the equation for beliefs (A15) and accumulation equations for $K_{C,t+1}$, and $K_{H,t+1}$.

The individual's problem, as approximated above, may be summarized as follows (where we drop age subscripts when no confusion arises and express trending variables as detrended):

$$\upsilon_{a,t}\left(\mu_t^{AG}, b_{F,t}, b_{F,t+1}, \mu_t^i\right) = \max_{h_{t+1}^i, \theta_{t+1}^i, b_{t+1}^i} U(c_t^i, h_t^i) + \beta\pi_i E_t[\upsilon_{a+1,t+1}\left(\mu_{t+1}^{AG}, b_{F,t}, b_{F,t+1}, \mu_{t+1}^i\right)] \quad s.t. \tag{A25}$$

The above problem is solved subject to (5) and (6), if the individual of working age, and subject to the analogous versions of (5) and (6), (using pension income in place of wage income), if the individual is retired. The problem is also solved subject to an evolution equation for the state space:

$$\mu_{t+1}^{AG} = \Gamma^{(n)}(\mu_t^{AG}, Z_{t+1}).$$

$\Gamma^{(n)}$ is the system of forecasting equations that is obtained by stacking all the beliefs from (A15) and accumulation equations into a single system. This step requires us to make an initial guess for $A^{(0)}$ in equations (A1)-(A4). This dynamic programming problem is quite complex numerically because of a large number of state variables but is otherwise straightforward. Its implementation is described below.

The collateral constraint faced by each household implies that net worth is non-negative so that accidental bequests are non-negative. If household i dies in period t then his net worth at death, left accidentally, is equivalent to the amount inherited by the newborn who replaces the dead individual. We allow the newborn to make an optimal portfolio choice over risky assets, bonds, and housing for how the bequeathed wealth is allocated in the first period of life. Favilukis, Ludvigson, and Van Nieuwerburgh (2008) study a model in which, in addition to these accidental bequests, some small fraction of households leave intentional bequests, driven by a bequest motive in their value functions.

Next we simulate the economy for a large number of individuals using the policy functions from the dynamic programming problem. Using data from the simulation, we calculate (A1)-(A4) as linear functions of $\widetilde{\varkappa}_t$ and an initial guess $A^{(0)}$. In particular, for every Z_t and Z_{t+1} combination we regress (A1)-(A4) on K_t, S_t, H_t, p_t^H, and q_t. This is used to calculate a new $A^{(n)} = A^{(1)}$ which is used to re-solve for the entire equilibrium. We continue repeating this procedure, updating the sequence $\left\{A^{(n)}\right\}$, $n = 0, 1, 2, ...$ until (1) the coefficients in $A^{(n)}$ between successive iterations is arbitrarily close, (2) the regressions have high R^2 statistics, and (3) the equilibrium is invariant to the inclusion of additional state variables such as additional lags and/or higher order moments of the cross-sectional wealth and housing distribution.

During the simulation step, an additional numerical complication is that two markets (the housing and bond market) must clear each period. This makes p_t^H and q_t convenient state variables: the individual's policy functions are a response to a menu of prices p_t^H and q_t. Given values for $Y_{H,t}$, $H^i_{a+1,t+1}$, $H^i_{a,t}$, $B^i_{a,t}$ and B_t^F form the simulation, and given the menu of prices p_t^H and q_t and the beliefs (A15), we then choose values for p_{t+1}^H and q_{t+1} that clear markets in $t+1$. The initial allocations of wealth and housing are set arbitrarily to insure that prices in the initial period of the simulation, p_1^H and q_1, clear markets. However, these values are not used since each simulation includes an initial burn-in period of 150 years that we discard for the final results.

The procedure just described requires a numerical solution to the individual's problem, a simulation using that solution for a large number of agents, and then a repetition (many times) of this procedure using the updated coefficients in $A^{(n)}$. The continuum of individuals born each period is approximated by a number large enough to insure that the mean and volatility of aggregate variables is not affected by idiosyncratic shocks. We check this by simulating the model for successively larger numbers of individuals in each age cohort and checking whether the mean and volatility of aggregate variables changes. We have solved the model for several different numbers of agents. For numbers ranging from a total of 2,500 to 25,000 agents in the population we found no significant differences in the aggregate allocations.

The R^2 statistics for the four equations (A1)-(A4) are (.999, .997, .999, .996), respectively. These R^2 are for 2,500 individuals. We found that successively increasing the number of individuals (beyond 2,500) successively increases the R^2 without affecting the equilibrium allocations or prices. However, we could not readily increase the number of agents beyond 25,000 because attempts to do so exceeded the available memory on a workstation computer. Our interpretation of this finding is that the equilibrium is unlikely to be affected by an approximation using more agents, even though doing so could result in an improvement in the R^2.

Numerical Solution to Individual's Dynamic Programming Problem

We now describe how the individual's dynamic programming problem is solved.

First we choose grids for the continuous variables in the state space. That is we pick a set of values for w^i, h^i, k, h, S, p^H, and q. Because of the large number of state variables, it is necessary to limit the number of grid points for some of the state variables given memory/storage limitations. We found that having a larger number of grid points for the individual state variables was far more important than for the aggregate state variables, in terms of the effect it had on the resulting allocations. Thus we use a small number of grid points for the aggregate state variables but compensate by judiciously choosing the grid point locations after an extensive trial and error experimentation designed to use only those points that lie in the immediate region where the state variables ultimately reside in the computed equilibria. As such, a larger number of grid points for the aggregate state variables was found to produce very similar results to those reported using only a small number of points. We pick 26 points for w_t^i, 13 points for h_t^i, three points for k_t, h_t, S_t, p_t^H, $b_{F,t}$, $b_{F,t+1}$ and four points for q_t. The grid for w^i starts at the borrowing constraint and ends far above the maximum wealth reached in simulation. This grid is very dense around typical values of financial wealth and is sparser for high values. The housing grid is constructed in the same way.

Given the grids for the state variables, we solve the individual's problem by value function iteration, starting for the oldest (age A) individual and solving backwards. The oldest individual's value function for the period after death is zero for all levels of wealth and housing. Hence the value function in the final period of life is given by $v_A = \max_{h_{t+1}^i, \theta_{t+1}^i, b_{t+1}^i} U(c_A^i, h_A^i)$ subject to the constraints above for (A25). Given v_A (calculated for every point on the state space), we then use this function to solve the problem for a younger individual (aged $A-1$). We continue iterating backwards until we have solved the youngest individual's (age 1) problem. We use piecewise cubic splines (Fortran methods PCHIM and CHFEV) to interpolate points on the value function. Any points that violate a constraint are assigned a large negative value.